THE
SYSTEM OF THE WORLD.

BY

P. S. LAPLACE,

MEMBER OF THE NATIONAL INSTITUTE OF FRANCE.

TRANSLATED FROM THE FRENCH

BY

J. POND, F.R.S.

IN TWO VOLUMES.

VOL. II.

LONDON:
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1809.

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OF THE THEORY OF UNIVERSAL GRAVITATION.

Opinionum commenta delet dies, naturæ judicia confirmat.
CIC. DE NAT. DEOR.

HAVING, in the preceding Books, explained the laws of the celestial motions, and those of the action of forces producing motion, we have now to compare them together, to learn what forces animate the solar system, to arrive without the assistance of any hypothesis, but by strict geometrical reasoning, at the principle of universal gravitation from which they are derived. It is in the celestial regions that the laws of mechanics are observed

with the greatest precision; on the earth so many causes tend to complicate their result, that it is very difficult to unravel them, and still more difficult to submit them to calculation. But the bodies of the solar system, separated by immense distances and subject to the action of a principal force, whose effect is easily calculated, are not disturbed in their respective motions by forces sufficiently considerable to prevent us from including under general formulæ, all the changes which a succession of ages has produced or may hereafter produce in the system. There is no question here of vague causes, which cannot be submitted to analysis, and which the imagination modifies at pleasure to accommodate them to the phenomena. The law of universal gravitation has this inestimable advantage, that it may be reduced to calculation, and by a comparison of its results with observation, it presents the most certain method of verifying its existence. We shall see that this great law of nature represents all the celestial phenome-

na even in their minutest details, that there is not one single inequality of their motions which is not derived from it, with the most admirable precision, and that it explains the cause of several singular motions, just perceived by astronomers, and which were either too complicated or too slow for them to recognize their law. Thus, so far from having to fear that new observations will disprove this theory, we may be assured before-hand, that they will only confirm it more and more; and we may be assured that its consequences are equally certain as if they actually had been observed. The most profound geometry was indispensable to establish these theories: I have collected them in my Treatise of Celestial Mechanics. I shall confine myself here to present the principal results of this work, indicating the steps that led to them, and explaining the reasons as far as can be done without the assistance of analysis.

CHAP. I.

Of the Principle of Universal Gravitation.

OF all the phenomena of the solar system, the elliptic motion of the planets and of the comets seems the most proper to conduct us to the general law of the forces by which they are animated. Observation has shewn that the areas described by the radii vectores of the planets and comets about the Sun are proportional to the times. Now we have seen in Chap. II. of the preceding Book, that for this to take place, the force which deflects the path of these bodies from a right line must constantly be directed towards the origin of their radii vectores. The tendency of the planets and comets to the Sun is therefore a necessary consequence of the proportionality of these areas to the times in which they are described.

To determine the law of this tendency, let us suppose the planets moved in circular orbits, which supposition does not greatly differ from the truth. The squares of their real velocities will then be proportional to the squares of the radii of these orbits divided by the squares of the times of their revolutions. But by the law of Kepler the squares of these times are to each other as the cubes of their radii. The squares of the velocities are therefore reciprocally as these radii. It has been shewn above that the central forces of several bodies moving in circular orbits, are as the squares of the velocities, divided by the radii of the circumferences described; the tendency therefore of the planets to the Sun is, reciprocally, as the squares of the radii of their orbits supposed circular. This hypothesis, it is true, is not rigorously exact, but the constant relation of the squares of the times to the cubes of the greater axes of their orbits, being independant of their excentricities, it is natural to think it would subsist also in the

case of the orbits being circular. Thus, the law of gravity towards the Sun varying reciprocally as the square of the distance is clearly indicated by this relation, analogy leads us to suppose that this law, which extends from one planet to another, subsists equally for the same planet at its different distances from the Sun, and its elliptic motion confirms this beyond a doubt. To comprehend this, let us follow this motion from the departure of the planet from its perihelion: its velocity is then at its maximum, and its tendency to recede from the Sun, surpassing its gravity towards it, its radius vector augments and forms an obtuse angle with the direction of its motion. The force of gravity towards the Sun decomposed according to this direction, continually diminishes the velocity, till it arrives at the aphelion; at this point the radius vector becoming perpendicular to the curve its velocity is a minimum, and its tendency to recede from the Sun being less than its gravity towards it, the planet will approach it describing

the second part of its ellipse. In this part the gravity towards the Sun increases its velocity in the same manner as it before decreased it, and the planet will arrive at its perihelion with its primitive velocity, and recommences a new revolution similar to the first. Now the curvature of the ellipse at the aphelion and perihelion being the same, the radii of curvature are the same, and consequently the centrifugal forces of these two points are as the squares of the velocities. The sectors described in the same time being equal, the aphelion and perihelion velocities are reciprocally as the corresponding distances of the planet from the Sun; the squares of these velocities are therefore reciprocally as the squares of these same distances; but at the perihelion and aphelion distances, the centrifugal forces in the osculatory circumferences are evidently equal to the gravity of the planet towards the Sun, which is therefore in the inverse proportion to the squares of these distances Thus the theorems of Huygens on the cen-

trifugal force were sufficient to demonstrate *the tendency of the planets towards* the Sun: for it is highly probable that this law, which extends from one planet to another, and which is verified in the same planet at its aphelion and perihelion, extends also to every part of the planetary orbit, and at the same time to every distance from the Sun. But to establish it in an incontestable manner, it was requisite to determine the general expression of the force which, directed towards the focus of an ellipse, obliges a projectile to describe that curve. And it was Newton who demonstrated *that this force was reciprocally* as the square of the radius vector. It was essential also to demonstrate rigorously that the force of gravity, towards the Sun, only varies in one planet from that of another from their different distances from it.

This great geometrician shewed, that this followed necessarily from the law of the squares of the periodic times being reciprocally as the cubes of the distance. Supposing, therefore, all the planets in

repose at the same distance from the Sun, and abandoned to their gravity towards its centre, they would descend from the same height in equal times; this result should likewise extend to the comets, notwithstanding the greater axes of their orbits are unknown, for we have seen in the second Book that the magnitude of the areas described by their radii vectores, supposes the law of the squares of their periodic times proportional to the cubes of their axes.

A general analysis, which embraces every possible result from a given law, shews us that not only an ellipse but any other conic section may be described by virtue of the force which retains the planets in their orbits; a comet may therefore move in an hyperbola, but then it would only be once visible, and would after its apparition recede from the limits of the solar system to approach other suns, which it would again abandon, thus visiting the different systems that are scattered in the immensity of the heavens. It is probable, considering

the infinite variety of nature, that such bodies exist. Their apparition should be a very rare occurrence; the comets we usually observe, are those which, having closed orbits, return at the end of intervals more or less considerable, into the regions of space that are in the vicinity of the Sun. The satellites tend also perpetually to the Sun. If the Moon was not subject to its action, instead of describing an orbit almost circular round the earth, it would soon finish by abandoning it; and if this satellite and those of Jupiter were not sollicited towards the Sun, according to the same law as the planets, sensible inequalities would result in their motions, which have not been recognized by observation. The planets, comets, and satellites are therefore subject to the same law of gravity towards the Sun. At the same time that the satellites move round their planet, the whole system of the planet and its satellites is carried by a common motion and retained by the same force, round the Sun. Thus the relative

motion of the planet and its satellites, is nearly the same as if the planet was at rest, and not acted on by any external force.

We are thus conducted without the aid of hypothesis, by a necessary consequence of the laws of the celestial motions, to consider the Sun as the centre of a force, which, extending infinitely into space, diminishes as the square of the distance increases, and which attracts all bodies that are in the sphere of its activity. Every one of the laws of Kepler discovers a property of this attractive force. The law of the areas proportional to the times, shews us that it is constantly directed towards the centre of the Sun; the elliptic orbits of the planets shew that this force diminishes as the square of the distance increases; finally, the laws of the squares of the periodic times proportional to the cubes of the distance, demonstrate that the gravity of all the planets towards the Sun is the same at equal distances; we shall call this gravity *the solar attraction* when

we speak of it as relative to the centre of the Sun towards which it is directed; for without knowing the cause, we may by one of those conceptions, common to geometricians, suppose an attractive power residing in the centre of the Sun.

The errors to which observations are liable, and the small alterations in the elliptic motion of the planets, leave a little uncertainty in the results which we have just deduced from the laws of motion; and it may be doubted if the solar gravity diminishes exactly in the inverse ratio of the square of the distance. But a very small variation in this law, would produce a very sensible difference in the motions of the perihelia of the planetary orbits. The perihelion of the terrestrial orbit, would have an annual motion of * 200″ if we only increased by one ten-thousandth part, the power of the distance to which the solar gravity is reciprocally proportional; this motion is only † 36″ 4, according to observation, and of this we shall hereafter see

the cause. The law of the square of the distance, is then at least extremely near; and its extreme simplicity should induce us to adopt it, as long as observations do not compel us to abandon it. At the same time we must not measure the simplicity of the laws of nature by our facility of conception; but when those which appear to us the most simple, accord perfectly with observations of the phenomena, we are justified in supposing them rigorously exact.

The gravity of the satellites towards the centre of their planet, is the necessary consequence of the proportionality of the areas described by their radii vectores to the times, and the law of the diminution of this force, according to the square of the distance, is indicated by the ellipticity of their orbits. But this ellipticity is hardly to be perceived in the orbits of the satellites of Jupiter, Saturn, and Uranus, which renders the law of the diminution of the force difficult to ascertain by the motion of any one single satellite; but the constant ratio of the squares of the times of their

revolutions, with the cubes of their distances, indicates it beyond a doubt, by demonstrating, that from one satellite to another, the gravity towards the planet is reciprocally as the square of the distance from its centre.

This proof is wanting for the earth, it having but one satellite, but it may be supplied by the following considerations.

The force of gravity extends to the summits of the highest mountains, and the small diminution which it there experiences, does not permit us to doubt, but that at still greater altitudes it would also be sensible. Is it not natural to extend this to the Moon, and to suppose that the force which retains it in its orbit, is its gravity towards the earth, in the same manner as the solar gravity retains the planets in their orbits round the Sun? For in fact these two forces seem to be of the same nature: they both of them penetrate the most intimate parts of matter, animating them with the same velocities; for we have seen that the solar gravity sollicits equally all bodies placed at

equal distances from the Sun, and that the terrestrial gravity also causes all bodies to fall through the same height in equal times.

A heavy body forcibly projected horizontally from a great height, falls on the earth at a great distance, describing a curve which is sensibly parabolic, it will fall still farther if the force is greater; and supposing it about seven thousand metres in a second, it would not fall to the Earth, but would circulate round it like a satellite, setting aside the resistance of the air. To form a moon of this projectile, it must be taken to the height of that body, and there receive the same motion of projection.

But what compleats the demonstration of the identity of the moon's tendency towards the earth with gravity, is that, to obtain this tendency, it is sufficient to diminish the terrestrial gravity according to the general law of the variation of the attractive force of the celestial bodies. Let us enter into the details that are suitable to the importance of this subject.

The force which at every instant deflects the Moon from the tangent of her orbit, causes it to describe, in one second, a space equal to the versed sine of the arc which it describes in that time; since this sine is the quantity that the Moon, at the end of a second, deviates from the direction it had in the beginning. This quantity may be determined by the distance of the Earth, inferred from the lunar parallax, in parts of the terrestrial radius; but to obtain a result independent of the inequalities of the Moon, we must take for the mean parallax that part of it which is independent of these inequalities. This part is according to observation * 10541″, relatively to the radius drawn from the centre of gravity of the earth, to the parallel of which the square of the sine of the latitude is equal to $\frac{1}{3}$. We select this parallel, because the attraction of the Earth on the points corresponding to its surface is, at the distance of the Moon, very nearly equal to the mass of the Earth,

* 56′ 51″ 9.

divided by the square of the distance from its centre of gravity. The radius drawn from a point of this parallel to the centre of gravity of the Earth is 6369374 metres, from whence it may be computed that the force which sollicits the Moon towards the Earth, causes it to fall $0^{me}.00101727$ in one second of time. It will be shewn hereafter, that the action of the Sun diminishes the lunar gravity $\frac{1}{358}$th part. The preceding height must therefore be augmented $\frac{1}{358}$th part, to render it independent of the action of the Sun; it then becomes $0^{me}.00102011$. But in its relative motion round the Earth, the Moon is sollicited by a force equal to the sum of the masses of the Earth and Moon, divided by the square of their mutual distance; therefore to obtain the height which the Moon would fall through in one second by the action of the Earth alone, the preceding space must be diminished in the ratio of the mass of the Earth to the Sun, of the masses of the Earth and Moon. But by the phenomena of the tides, it appears that the mass of the Moon is equal $\frac{1}{58,7}$ of

that of the earth, multiplying therefore this space by $\frac{58{,}7}{59{,}7}$, we have $0^{me}.00100300$ for the height which the Moon falls through in one second by the action of the Earth.

Let us now compare this height with that which results from observation made on the pendulum. Under the parallel above mentioned, the length of the pendulum vibrating seconds is (by Chapter XII. Book I.) equal to $3^{me}.65706$. But on this parallel, the attraction of the Earth is less than the force of gravity by $\frac{2}{3}$ of the centrifugal force due to the motion or notation of the Earth at the equator; and this force is $\frac{1}{288}$th part of that of gravity; the preceding space must therefore be augmented $\frac{1}{432}$d part, to have the space due to the action of terrestrial gravity alone, which on this parallel is equal to the mass divided by the square of the terrestrial radius, we shall therefore have $3^{me}.66553$ for this space. At the distance of the Moon it should be diminished in the ratio of the square of the radius of the terrestrial spheroid to the square of the distance of the

Moon: for this it is sufficient to multiply it by the square of the tangent of the lunar parallax, or † 10541′, this will give 0^{mc}. 00100483 for the height which the Moon should fall through in one second by the attraction of the Earth. This quantity derived from experiments on the pendulum, differs very little from that which results from direct observation of the lunar parallax; to make them coincide, it is sufficient to diminish the parallax * 6″, and to reduce it to ‡ 10535″. This is the parallax resulting from the theory of gravity, differing only $\frac{1}{1600}$th part from that derived from direct observation, to which I think it preferable, considering the exactness of the elements from which it is computed. It would be sufficient to diminish a little the mass of the Moon, to obtain by this theory of gravity the same parallax that is given by observation: but all the phenomena of the tides concur in giving to this satellite a mass more considerable, and

* 2″. † 56′ 55″ 2. ‡ 56′ 53″ 3.

such very nearly as we have used in the above computation. But however that may be, the small difference between the two parallaxes is within the limits of the errors of observation, and of the elements employed in the calculation. It is then certain, that the force which retains the Moon in its orbit is the terrestrial gravity diminished in proportion to the square of the distance. Thus the law of the diminution of gravity, which, in planets accompanied by several satellites, is proved by a comparison of their periodic times with their distances, is demonstrated for the Moon, by comparing its motion with that of projectiles at the surface of the Earth.

The observations of the pendulum made on the summits of mountains had already indicated this diminution of the terrestrial gravity; but they were insufficient to discover the law, because of the small height of the most elevated mountains, compared with the radius of the Earth: it was requisite to find a body very remote from us, as

the Moon, to render the law perceptible, and to convince us that the force of gravity on the Earth is only a particular case of a force which pervades the whole universe.

Every phenomenon throws new light and confirms the laws of nature. It is thus that the comparison of experiments on gravity with the lunar motion, shews us, that the origin of the distances of the Sun and of the planets in the calculation of their attractive forces, should be placed in their centres of gravity; for it is evident this takes place on the Earth, whose attractive force is of the same nature as that of the Sun and planets.

The Sun, and those planets which are accompanied by satellites, being thus endowed with an attractive force varying inversely as the square of the distance, a strong analogy leads us to attribute the same property to the other planets. The spherical figure common to all these bodies, indicates that their particles are united round their centers of gravity by a force which, at equal distances, equally sollicits

them towards these points; but the following considerations leave no doubt on this subject.

We have seen that if the planets and the comets were placed at the same distance from the Sun, their gravity towards it would be in proportion to their masses: now it is a general law in nature, that action and reaction are equal and contrary, all these bodies therefore react on the Sun, and attract in proportion to their masses; they are therefore endowed with an attractive force proportional to their masses, and inversely as the square of the distance. By the same principle the satellites attract the planets and the Sun according to the same law. This attractive property then is common to all the celestial bodies: it does not disturb the elliptic motion round the Sun, when we consider only their mutual action; for the relative motion of the bodies of a system are not changed by giving them a common velocity: by impressing therefore, in a contrary direction to the Sun and to the planet, the motion of the first of these

two bodies, and the action which it experiences on the part of the second, the Sun may be considered as immovable; but the planet will be sollicited towards it with a force reciprocally as the squares of the distance and proportional to the sum of the masses: its motion round the Sun will therefore be elliptic. And we see by the same reasoning that it would be so if the planet and Sun were carried through space, with a motion common to each of them. It is equally evident that the elliptic motion of a satellite is not disturbed by the motion of translation of its planet, nor would it be by the action of the Sun, if it was always exactly the same on the satellite and the planet. Nevertheless, the action of a planet on the Sun influences the length of its revolution, which is diminished as the mass of the planet is more considerable, so that the relation of the square of its periodic time to the cube of the major axis of its orbit, depends on its mass. But since this relation is nearly the same for all the planets, their masses must evidently be very small com-

pared with that of the Sun, which is equally free for the satellites with respect to their principal planets. This we may readily suppose from the smallness of their volumes.

The attractive property of the heavenly bodies does not only belong to them in a mass, but belongs to each of their particles. If the Sun only acted on the centre of the Earth, without attracting particularly every one of its particles, there would arise in the ocean oscillations infinitely more considerable, and very different from those which we observe. The gravity of the Earth therefore to the Sun is the result of the gravity of all its particles, which consequently attract the Sun in proportion to their respective masses; besides each body on the earth, tends towards its centre proportionally to its mass, it reacts therefore, on it, and attracts it in the same ratio. If that was not the case, and if any part of the Earth, however small, attracted another part without being attracted by it, the

virtue of the force of gravity, which is impossible.

The celestial phenomena compared with the laws of motion, conduct us therefore, to this great principle of nature, namely, *that all the particles of matter attract each other in proportion to their masses, and inversely as the squares of their distance.*

Already we may perceive in this univer-sal gravitation the cause of the perturbations to which the heavenly bodies are subject; for the planets and comets being subject to the action of each other, they must deviate a little from the laws of the elliptic motion, which they would otherwise exactly follow, if they only obeyed the action of the Sun. The satellites also deranged in their motions round their planets by their mutual action and that of the Sun, deviate a little from these laws.

We perceive, then, that the particles of the heavenly bodies, united by their attraction, should form a mass nearly spherical; and that the result of their action at the surface of the body, should produce

all the phenomena of gravitation. We see, moreover, that the motion of rotation of the celestial bodies should slightly alter their spherical figure, and flatten them at the poles; and then the resulting force of all their mutual actions not passing through their centres of gravity, should produce in their axes of rotation similar motions to those discovered by observation. Finally, we may perceive why the particles of the ocean, unequally acted on by the Sun and Moon, should have oscillations similar to the ebbing and flowing of the tides. But these different effects of the principle of gravitation, must be particularly developed to give it all the certainty of which physical truth is susceptible.

CHAP. II.

Of the Masses of the Planets, and of Gravity at their Surface.

It appears on the first view of the subject impossible to determine the respective masses of the Sun and planets, and to measure the height from which bodies fall in a given time, from the action of gravity at their surface. But the connection of truths with each other conducts us to results which appeared inaccessible, when the principle on which they depend was unknown. Thus the measure of the intensity of gravity at the surface of the planets is rendered practicable by the discovery of universal gravitation. Let us return to the theorems on centrifugal force given in the preceding book. The result derived from them is, that the gravity of a satellite towards its planet is to the gravity of the Earth towards the Sun, as the

divided by the square of the time of its sidereal revolution, is to the mean distance of the Earth from the Sun, divided by the square of a sidereal year. To reduce these gravities to the same distance from the bodies which produce them, they must be multiplied respectively by the squares of the radii of the orbits which they describe. And as at equal distances the masses are proportional to their attractions, the mass of the Earth is to that of the Sun, as the cube of the mean radius of the orbit of the satellite, divided by the square of the time of its sidereal revolution, is to the cube of the mean distance of the Earth from the Sun, divided by the square of the sidereal year. Let this result be applied to Jupiter. For this purpose we shall observe that the mean radius of the orbit of the fourth satellite subtends at the mean distance of Jupiter from the Sun an angle of *1530′ 86, seen at the mean distance of

* 8′ 15″ 9.

the Earth from the Sun, this radius appears under an angle of * 7964″75; the radius of the circle contains † 636619″ 8: thus the mean radii of the orbits of the fourth satellite, and of the terrestrial orbit, are in the proportion of these two last numbers. The duration of the sidereal revolution of the fourth satellite is ‡ 16 6890, and the sidereal year is § 365^d 2564. Setting out from these data the mass of Jupiter is found to be $\frac{1}{1066.08}$, that of the Sun being represented by unity. To obtain greater exactness, it is necessary to augment by unity the denominator of this fraction, because the force which retains Jupiter in its relative orbit round the Sun, is the sum of the attractions of the Sun and of Jupiter. The mass of this planet then is $\frac{1}{1067.08}$. I have determined by the same method the masses of Saturn and of Uranus. That of the Earth may be calculated in the same manner, but the following

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* 43′ 0″ 5. † 57° 17′ 44′ 8.

method is yet more precise. If the mean distance of the Earth from the Sun be taken for unity, the arc described by the Earth in a second of time, will be the proportion of the circumference, to radius, divided by the number of seconds in the sidereal year, or by *36525638″ 4 ; dividing the square of this arc by the diameter, we shall get $\frac{1479565}{1020}$ for its versed sine, it is the quantity which the Earth falls towards the Sun during one second, in consequence of its relative motion round it. It has been seen, in the preceding chapter, that upon the terrestrial parallel, the sine of the latitude of which is $\frac{1}{3}$, the attraction of the Earth causes bodies to fall through the Earth $3^{me.}$ 66553 in one second. To reduce this attraction to the mean distance of the Earth from the Sun, it must be multiplied by the square of the sine of the solar parallax, and this product divided by the number of metres contained in this distance. Now the terrestrial radius, upon the parallel we are considering, is 6369374 me-

tres;dividing this number, therefore,by the sine of the solar parallax, or by * 27″ 2, we shall get the mean radius of the terrestrial orbit expressed in metres. It follows from hence that the effect of the Earth's attraction at the mean distance of this planet from the Sun, is equal to the product of the fraction $\frac{366553}{6369374}$ by the cube of the sine of 27″ 2; it is consequently equal to $\frac{4.48855}{1020}$: taking this fraction from $\frac{1479565}{1020}$ we shall have $\frac{1479605}{1020}$ for the effect of the Sun's attraction at the same distance. The masses of the Sun and Earth are therefore in the proportion of the numbers 1479560.5 and 4.48855 ; from whence it follows that the mass of the Earth is $\frac{1}{329630}$. If the parallax of the Sun is a little different from what we have admitted, the value of the mass of the Earth should vary as the cube of this parallax compared to that of 27″ 2.

The following determinations of the masses of those planets which have no sa-

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* 8″ 5.

tellites, have been obtained by the secular changes which the action of these bodies produces in the elements of the solar system. I have determined the masses of Venus and Mars from the secular diminution of the obliquity of the ecliptic, supposed to be * 154″ 3, and from the acceleration of the Moon's mean motion, fixing it at †34.36 for the first century, setting out from 1700. The mass of Mercury has been determined by its volume, supposing the densities of this planet and of the Earth inversely as their mean distances from the Sun. An hypothesis really very precarious, but which corresponds with sufficient exactness to the respective densities of the Earth, Jupiter and Saturn. It will be necessary to rectify all these values when time shall have demonstrated more correctly the secular variations in the celestial motions and orbits.

The masses of those planets which are accompanied by satellites should be also rectified by very precise observations of

the greatest elongation of the satellite from their planets, without neglecting the consideration of the ellipticity of their orbits.

Masses of the Planets, that of the Sun being taken as unity.

Mercury	$\frac{1}{2025810}$
Venus	$\frac{1}{383137}$
The Earth	$\frac{1}{329630}$
Mars	$\frac{1}{1846082}$
Jupiter	$\frac{1}{1067{,}09}$
Saturn	$\frac{1}{3359{,}40}$
Uranus	$\frac{1}{19504}$

The densities of bodies are proportional to their masses divided by their volumes, and when they are nearly spherical their volumes are as the cubes of their radii. Their densities therefore are as their masses divided by the cubes of their radii; but to obtain greater accuracy, that radius of a planet must be taken which corresponds to that parallel, the square of the sine of whose latitude is $\frac{1}{3}$, and which is equal to one third of the sum of the radius of the pole, added to twice the radius of the equator. It is thus

found that taking the mean density of the Sun for unity, those of the Earth, Jupiter, Saturn, and Uranus are 3.9393, 0.8601, 0.4951 and 1.1376.

We ought to observe that the error in the measures of the apparent diameter of the planets, and the irradiation, which we have not considered on account of the great difficulty of appreciating it, may influence these calculations very perceptibly. We shall again observe that the preceding value of the density of the Earth is independent of the solar parallax; for both its mass and volume, compared to the Sun, increase as the cube of this parallax.

The measures of the greatest elongations of satellites from their planets merit particularly the attention of observers, since on this depends the knowledge of the masses and densities of the planets. Newton has proposed a very simple method to divest the apparent diameter of the effect of irradiation. It consists in observing, during the night, the light of a lamp through

ance, and small enough only to suffer a part of the light to be visible. The brilliance of the light and the opening is to be diminished till the lamp appears exactly the same size and brightness as the planet; the proportion of the opening to the distance of the observer, will give with great precision the diameter of the planet. The appearances of Saturn's ring may be thus represented, the dimensions of the interior and exterior ring measured, concerning which irradiation produces so much uncertainty. To obtain the intensity of gravitation at the surface of the Sun and planets, let us consider that if Jupiter, and the Earth were exactly spherical, and deprived of their rotatory motion, gravity at their equators would be proportionate to the masses of these bodies, divided by the squares of their diameters; now at the mean distance from the Sun to the Earth, Jupiter's equatorial diameter is *626″26, and that of the Earth's equator is † 54″5;

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* 3′ 22. † 17″ 5.

representing then the weight of a body at the terrestrial equator by unity, the weight of this body transported to the equator of Jupiter would be 2,509, but this weight must be diminished by about a ninth from the effects of the centrifugal force due to the rotation of these planets. The same body would weigh 27,65 at the equator of the Sun, and falling bodies would describe one hundred metres in the first second of their descent.

CHAP. III.

Of the Perturbations of the Elliptic Motion of the Planets.

IF the planets only obeyed the action of the Sun, they would revolve round it in elliptic orbits, but they act mutually upon each other and upon the Sun, and from these various attractions there result perturbations in their elliptic motions, which are to a certain degree perceived by observation, and which it is necessary to determine to have exact tables of the planetary motions. The rigorous solution of this problem, surpasses at present the powers of analysis, and we are obliged to have recourse to approximations. Fortunately the smallness of the masses compared to the Sun, and the smallness of their excentricity and inclination of their orbits, affords

considerable facility to this object. It is still, however, sufficiently complicated, and the most delicate and intricate analysis is requisite to detect among the infinite number of inequalities to which the planets are subject, those which are sensible to observation and to assign their values.

The perturbations of the elliptic motion of the planets may be divided into two distinct classes. Those of the first class affect the elliptic motion of the planets, they increase with extreme slowness and are called *secular inequalities.* The other class depends on the configurations of the planets, both with respect to each other and to their nodes and perihelia, and being re-established every time these configurations become the same, they have been called *periodical inequalities* to distinguish them from secular inequalities, which are equally periodic but whose periods are much longer and independent of the mutual configurations of the planets.

The most simple manner of considering

these various perturbations, consists in imagining a planet moving according to the laws of the elliptic motion upon an ellipse whose elements vary by imperceptible gradations, and conceiving at the same time the true planet to oscillate round the imaginary planet in a small orbit, the nature of which must depend on its periodic inequalities. Thus its secular inequalities are represented by the imaginary planet, and its periodic inequalities by its motion round this same planet.

Let us first consider those secular inequalities which, by developing themselves, in the course of ages, should change at length both the form and position of the planetary orbits. The most important of these inequalities is that which may affect the mean motion of the planets. By comparing together the observations which have been made since the re-establishment of astronomy, the motion of Jupiter appears to be quicker and that of Saturn slower, than by a comparison of the same observations with those of the ancient astronomers:

from which it has been inferred that the first of these motions has accelerated, while the second has retarded from one century to another. And to take into account these variations, astronomers have introduced into their tables of planets, two secular equations increasing with the squares of the times, one additive to the motion of Jupiter, the other subtractive from that of Saturn. According to Halley the secular equation of Jupiter is *106″02 for the first century reckoned from 1700, the corresponding equation of Saturn is †156″94. It was natural to look for the cause of these equations in the mutual actions of these two planets, the most considerable of our system. Euler, who first directed his attention to this problem, found a secular equation, equal for both the planets, and additive to their mean motions, which is inconsistent with observation. Lagrange obtained a result which accorded more nearly with them. Other geometricians obtained other equations. Struck

with this difference, I examined again this subject, applying the greatest possible care to the investigation, and I arrived at the true analytical expression for the secular inequality of the mean motions of the planets. In substituting the numerical values, relative to Jupiter and Saturn in this expression, I was surprised to find that it became equal to nothing. I suspected that this was not peculiar to these planets, and that if this expression was put in the most simple form of which it was susceptible, (by reducing to the least possible number the different quantities which it contains by means of the relations which subsist between them) all its terms would destroy each other. Calculation confirmed this suspicion, and taught me that, in general, the mean motions of the planets and their distances from the Sun are invariable; at least when we neglect the fourth powers of the excentricities and inclinations of the orbits, and the squares of the perturbating masses, which is more than sufficient

for the actual purposes of astronomy. Lagrange has since confirmed this result, and shewn, by a beautiful method, that it is even true, when the powers and products of any order whatever of the excentricities and inclinations are taken into the calculation. Thus the variations of the mean motions of Jupiter and Saturn do not depend on their secular inequalities.

The permanency of the mean motion of the planets and of the greater axes of their orbits, is one of the most remarkable phenomena in the system of the world. All the other elements of the planetary ellipses are variable, all these ellipses approach to and depart insensibly from the circular form; their inclination to a fixed plane or to the ecliptic augments and diminishes, and their perihelia and nodes are continually changing their places. These variations which are performed with extreme slowness, arise from the mutual actions of the planets on each other, and require several centuries for their completion. They are nearly proportional to the times. They

have already become apparent by observation; we have seen, in the first Book, that the perihelion of the Earth's orbit has a direct annual motion of *36″7, and that its inclination to the equator diminishes every century †154″3. It was Euler that first investigated the cause of this diminution, which all the planets contribute to produce by the respective situation of the planes of their orbits. The ancient observations are not exact enough, and the modern are too near each other to fix the exact quantity of these great changes, nevertheless they combine to prove their existence, and to shew that their progress is the same as is conformable to the law of gravitation. If we knew exactly the masses of the planets, future observations might be anticipated, and the true values assigned to the secular inequalities of the planets; but we only know the masses of those planets which are accompanied by satellites, the masses of the others can only

be determined when the progress of time shall have fully developed the quantity of these inequalities from whence these masses are to be computed. We may then in imagination look back to the successive changes which the planetary system has undergone, and foretell those which future ages will offer to astronomers, and the geometrician will at once comprehend in his formulæ both the past and future state of the world. The table of Chap. V. of the second Book, contains the secular variation which results from the preceding masses which we have assigned to the planets.

Many interesting questions here present themselves to our notice. Have the planetary ellipses always been, and will they always be nearly circular? Among the number of the planets have any of them ever been comets whose orbits have gradually approached to the circular form by the mutual attractions of the other planets? Will the obliquity of the ecliptic continually diminish till at length it coincides with the equator, and the days and nights

become equal on the earth throughout the year? Analysis answers these questions, in a most satisfactory manner. I have succeeded in demonstrating that whatever be the masses of the planets, in as much as they all move in the same direction, in orbits of small excentricity, and little inclined to each other; their secular inequalities will be periodic, and contained within narrow limits, so that the planetary system will only oscillate about a mean state, from which it will deviate but by a very small quantity; the planetary ellipses therefore always have been, and always will be nearly circular, from whence it follows that no planet has ever been a comet, at least if we only calculate upon the mutual actions of the planetary system. The ecliptic will never coincide with the equator, and the whole extent of its variations will not exceed * three degrees.

The motions of the planetary orbits and of the stars will one day embarrass astronomers when they attempt to compare pre-

* 2° 42.

cise observations separated by long intervals of time; already this difficulty begins to be manifest; it would be interesting therefore to find some plane that should remain invariable, that, is constantly parallel to itself during all these changes. There fortunately exists such a one, which possesses this remarkable property, to which the orbits of the planets may be referred, just as naturally as the motion of a system of bodies to its centre of gravity. This plane may easily be determined by the following rule.

If, at any instant of time whatever, and upon any plane passing through the centre of the Sun, we draw straight lines to the ascending nodes of the planetary orbits referred to this plane, and if we take on these lines, reckoning from the centre of the Sun, lines equal to the tangents of the inclinations of these orbits to this plane; and if at the extremities of these lines we suppose masses equal to the masses of the planets multiplied respectively into the square roots of the parameters of their orbits, and by the cosines of their inclinations; and lastly, if

we determine the centre of gravity of this new system of masses, then the straight line drawn from the centre of the Sun to this point will be the tangent of the inclination of the invariable plane to the assumed plane; and continuing this line to the heavens, it will there mark its ascending node.

Whatever changes the succession of ages may produce in the planetary orbits, and whatever be the plane to which they are referred, the plane determined by this rule will always be the same. It is true its position depends on the masses of the planets; but those which have satellites have the greatest influence on its position, and the masses of the others will soon be sufficiently known to determine it with exactness. In adopting the preceding values of the planets, and the elements of their orbits, as given in Chap. V. Book II. we find that the longitude of the ascending nodes of the invariable plane was * 114°3877 at the commencement of 1750, and at

was * 1° 719. In this computation we have neglected the comets, which nevertheless ought to enter into the determination of the invariable plane, since they make part of the solar system. It would be easy to include them in the preceding rule, if their masses and the elements of their orbits were known. But in our present ignorance of the nature of these objects, we suppose their masses too small to influence the planetary system, and this is the more probable, since the theory of the mutual attraction of the planets suffices to explain all the inequalities observed in their motions. But if the action of the comets should become sensible in length of time, it should principally affect the position of the plane, which we suppose invariable, and in this new point of view the consideration of this plane will still be useful, if the variations of this plane could be recognised, which would be attended with great difficulties.

The theory of the secular and periodic

inequalities of the motions of the planets, founded on the law of universal gravitation, has given to our astronomical tables a precision which proves the correctness and utility of this theory. By its means the solar tables which before deviated† two minutes from the observations, have acquired the same precision as the observations themselves. It is particularly in the motions of Jupiter and Saturn that these inequalities are most sensible, but they present themselves under a form so complicated, and the length of their periods is so considerable, that it would have required several ages to have determined their law by observations alone, which has in this instance been anticipated by theory.

After having established the invariability of the mean motions of the planets, I suspected that the alterations observed in the mean motions of Jupiter and Saturn proceeded from the action of comets. Lalande had remarked in the motion of Saturn, irregularities which did not appear

† 1′ 4″.

to depend on the action of Jupiter: he found its returns to the vernal exquinox more rapid than its returns to the autumnal equinox, although the positions of Jupiter and Saturn both to each other and to their aphelia, were nearly the same. Lambert likewise observed that the mean motion of Saturn which seemed to diminish from century to century by the comparison of antient with modern observations, appeared on the contrary to accelerate by the comparison of modern observations with each other, at the same time that Jupiter presented phenomena exactly contrary. All this seemed to indicate that causes independent of the action of Jupiter and Saturn on each other had altered their motions. But on mature reflections, the order of the variations observed in the mean motions of these planets, appeared to me to agree so well with the theory of their mutual attraction, that I did not hesitate to reject the hypothesis of a foreign cause.

It is a remarkable result of the mutual tion of the planets on each other, that

if we only consider the inequalities which have very long periods, the sum of the masses of every planet, divided respectively by the greater axis of their orbits, is always pretty nearly constant. From this it follows that the squares of the mean motions being reciprocally as the cubes of these axes, if the motion of Saturn is retarded by the action of Jupiter, that of Jupiter should be accelerated by the action of Saturn, which is conformable to observation. I perceived, moreover, that the law of these variations was the same as corresponded to the preceding theory. In supposing with Halley the retardation of Saturn to be *256″94 for the first century, reckoned from 1700, the corresponding acceleration of Jupiter should be † 109″80, and Halley found ‡ 106″02 by observation. It was therefore very probable that the variations observed in the mean motions of Jupiter and Saturn, were the effects of their mutual action; and since it is certain that this

* 1′ 23″. † 35″ 5. ‡ 34″ 3.

action cannot produce any inequality either constantly increasing or periodic, but of a period *independent* of the configuration of these planets, and that it cannot effect in it any irregularities but what are relative to this configuration, it was natural to think that there existed in their theory a considerable inequality of this kind, of a very long period, and which was the cause of these variations.

The inequalities of this kind, although very small and almost insensible in differential equations, augment considerably in the integrations, and may acquire very great values in the expressions of the longitudes of the planets. I easily recognized the existence of similar inequalities in the differential equations of the motions of Jupiter and Saturn. These motions become very nearly commensurable; and five times the mean motion of Saturn is very nearly equal to twice that of Jupiter: from which I concluded that the terms which have for their argument five times the mean longitude of Saturn, minus twice that of Jupiter, might by integration

become very sensible, although multiplied by the rules and products of three dimensions of the excentricities and inclinations of the orbits. I considered therefore that these terms were the probable cause of the variations observed in the mean motions of these planets. The probability of this cause, and the importance of the object, determined me to undertake the laborious calculation, necessary to determine this question. The result of this calculation fully confirmed my conjecture; and it appeared, that in the first place there exists in the theory of Saturn a great inequality of * 9027″7 at its maximum, and of which the period is 917¾ years; and, secondly, that the motion of Jupiter is subject to a similar inequality, whose period and law are the same, but its amount is only † 3856″5. It is to these two inequalities, formerly unknown, that we must attribute the apparent retardation of Saturn, and apparent acceleration of Jupiter. These phenomena attain-

* 48′ 44″ 9. † 20′ 49″ 5.

ed their maximum about the year 1560; since this epoch, their mean apparent motions have approximated to their true mean motions, and they were equal in 1790. This explains the reason why Halley, in comparing the antient and modern observations, found the mean motion of Saturn slower, and that of Jupiter more rapid than by the comparison of modern observations with each other, instead of which these last indicated to Lambert an acceleration in the motion of Saturn, and a retardation in that of Jupiter. And it is very remarkable that the quantities of these phenomena, deduced from observation alone by Halley and Lambert, are very nearly the same as result from the two great inequalities which I have just mentioned. If astronomy had been revived four centuries and a half later, the observations would have presented the contrary phenomena. The mean motions which the astronomy of any people have assigned to Jupiter and Saturn, may afford us information concerning the time of its foundation. Thus

it appears that the Indian astronomers determined the mean motions of these planets, in that part of the period of the preceding inequalities, when the motion of Saturn was the slowest, and that of Jupiter the most rapid. Two of their principal astronomical epochs, the one 3102 A.C. the other 1491 A.C. answer nearly to this condition. The nearly commensurable relation that exists in the motions of Jupiter and Saturn, occasions other very perceptible inequalities, the most considerable of which affects the motion of Saturn; it would be entirely confounded in the equation of the centre, if twice the mean motion of Jupiter was exactly equal to five times that of Saturn. The difference observed in this century in the intervals of the returns of Saturn to the equinoxes both of spring and autumn, arises principally from this cause.

In general, when I had recognised these various inequalities, and examined more carefully than had been done before, those which had been submitted to calculation, I found that all the observed phenomena

of the motions of these two planets adapted themselves naturally to the theory; they before had seemed to form an exception to the law of universal gravitation; they are now become one of the most striking examples of its truth. Such has been the fate of this brilliant discovery, that every difficulty that has arisen has only furnished a new subject of triumph for it, which is the most indubitable characteristic of the true system of nature.

I cannot in this place refrain from making a comparison of the real effects of this relation between the mean motion of Jupiter and Saturn, with those which astrology had attributed to it. In consequence of this relation, if the conjunction of the two planets arrives in the first point of Aries, it will in twenty years afterwards take place in Sagittarius, and in twenty years afterwards in Leo, it will continue to take place in these three signs for nearly two hundred years. In the same manner in the next two hundred years, it will go through the signs Taurus, Capricornus, and Virgo. In the

next two hundred years it will proceed through the signs Gemini, Aquarius, and Libra; and finally, in the last two hundred years it will describe the remaining signs Cancer, Pisces, and Scorpio; after which it will again begin with the sign Aries as before. From hence arises a great year, each season of which is equal to two centuries. They attributed different temperatures to the different seasons of this year, as likewise to the signs which belonged to them. The assemblage of these three signs was called a *trigon*. The first trigon was that of Fire, the second of Earth, the third of Air, and the fourth of Water.—We may easily imagine that astrology made great use of these trigons, which even Kepler himself describes with great exactness, in several of his works: but it is very remarkable that sound astronomy in dissipating the imaginary influence that was supposed to attend this relation in the motion of the two planets, should have recognised in the harmony of this relation, the

source of the greatest perturbations of the planetary system.

The planet Uranus, though lately discovered, offers already incontestable indications of the perturbations which it experiences from the action of Jupiter and Saturn. The laws of elliptic motion do not exactly satisfy its observed positions, and to represent them its perturbations must be considered. Their theory, by a very remarkable coincidence, places it in the years 1769, 1756, and 1690, in the same points of the heavens, where Monnier, Mayor, and Flamstead, had determined the position of three stars, which cannot be found at present: this leaves no doubt of the identity of these stars with the new planet.

CHAP. IV.

Of the Perturbations in the Elliptic Motion of Comets.

The action of the planets produces, in the motion of comets, inequalities which are principally sensible in the intervals of their returns to the perihelion. Halley having remarked that the elements of the orbits of the comets observed in 1531, 1607, and 1682, were nearly the same, concluded that they belonged to the same comet which in the space of 151 years had made two revolutions. It is true that the period of the first revolution is thirteen months longer than the second. But this great astronomer thought, and with reason, that the attraction of the planets, particularly of Jupiter and Saturn, might have occasioned this difference, and after a vague estimation

of this action for the course of the following period, he judged that it should retard the return of the comet, and he fixed it for the end of 1758, or the commencement of 1759. This prediction was too important in itself, and too intimately connected with the theory of universal gravitation, not to excite the curiosity of all those who were interested in the progress of the sciences; for about this time geometricians were very much engaged in extending the application of this theory. During the whole year of 1757, astronomers looked for this comet; and Clairaut, who had been one of the first to solve the problem of the three bodies, applied his solution to the determination of the inequalities which the comet had sustained by the action of Jupiter and Saturn. The 14th November, 1758, he announced in the academy of sciences, that the interval of the return of the comet to its perihelion, would be 618 days longer in the present actual period than in the former one, and that consequently the comet would pass its perihelion about the middle of April

1759. He observed, at the same time, that the small quantities neglected in this approximate calculation, might advance or retard this term a month. That moreover, a body which passes into regions so remote, and which escapes our sight during such long intervals, may be subject to the action of forces entirely unknown, as the attraction of other comets, or even of some planet, whose distance is too great to be ever visible to us. This philosopher had the satisfaction of seeing his prediction accomplished; the comet passed its perihelion the 12th March, 1759, within the limits of the errors of which he thought his results susceptible. After a new revision of his calculations, Clairaut fixed this passage at the 4th of April, and he would have brought it to the 25th March, if he had employed the mass of Saturn, such as is given in chap. II.; that is, within thirteen days of the actual observation. This difference will appear very small, if we consider the great number of quantities neglected, and the influence which the planet

Uranus might produce, whose existence was at that time unknown.

Let us remark, for the honour of the human understanding, that this comet, which in this century only excited the curiosity of astronomers and mathematicians, had been regarded in a very different manner, four revolutions before, when it appeared in 1456. Its long tail spread consternation over all Europe, already terrified by the rapid success of the Turkish arms, which had just destroyed the great empire. Pope Callixtus, on this occasion, ordered a prayer, in which both the comet and the Turks were included in one anathema.

In those times of ignorance, mankind were far from thinking, that the only mode of questioning nature is by calculation and observation: according as phenomena succeeded with regularity or without apparent order, they were supposed to depend either on final causes or on chance; and whenever any happened which seemed out of the natural order, they were considered as so many signs of the anger of heaven.

But these imaginary causes have successively given way to the progress of knowledge, and will totally disappear in the presence of sound philosophy, which sees nothing in them, but expressions of the ignorance of the truth.

To the terrors which the apparition of comets then inspired, succeeded the fear, that of the great number which traverse the planetary system in all directions, one of them might overturn the earth.

They pass so rapidly by us, that the effects of their attraction are not to be apprehended. It is only by striking the earth that they can produce any disastrous effect. But this circumstance, though possible, is so little probable in the course of a century, and it would require such an extraordinary combination of circumstances for two bodies, so small in comparison with the immense space they move in, to strike each other, that no reasonable apprehension can be entertained of such an event.

Nevertheless, the small probability of this circumstance may, by accumulating

during a long succession of ages, become very great. It is easy to represent the effect of such a shock upon the earth: the axis and motion of rotation changed, the waters abandoning their antient position, to precipitate themselves towards the new equator; the greater part of men and animals drowned in a universal deluge, or destroyed by the violence of the shock given to the terrestrial globe; whole species destroyed; all the monuments of human industry reversed: such are the disasters which a shock of a comet would produce.

We see then why the ocean has abandoned the highest mountains, on which it has left incontestible marks of its former abode: we see why the animals and plants of the south may have existed in the climates of the north, where their relics and impressions are still to be found: lastly, it explains the short period of the existence of the moral world, whose earliest monuments do not go much farther back than three thousand years. The human race reduced to a small number of individuals,

in the most deplorable state, occupied only with the immediate care for their subsistence, must necessarily have lost the remembrance of all sciences and of every art; and when the progress of civilization has again created new wants, every thing was to be done again, as if mankind had been just placed upon the earth. But whatever may be the cause assigned by philosophers to these phenomena, we may be perfectly at ease with respect to such a catastrophe during the short period of human life.

But man is so disposed to yield to the dictates of fear, that the greatest consternation was excited at Paris, and communicated to the provinces in 1773, by a memoir of Lalande, in which he determined, of those comets which had been observed, the orbits that most nearly approached the earth; so true it is, that error, superstition, vain terrors, and all the evils of ignorance are ever ready to start up, when the light of science is unfortunately extinguished.

The action of comets upon the solar

system has been hitherto insensible, which seems to indicate that their masses are inconsiderable. It is possible, however, that the minute errors of our best tables depend upon it. An exact theory of the perturbation of the planets, compared with very precise observations, is the only means of ascertaining this point, so important to the system of the world.

CHAP. V.

Of the Perturbations of the Motion of the Moon.

The Moon is attracted at the same time by the Sun and by the Earth, but its motion round the Earth is only disturbed by the difference of the action of the Sun upon these two bodies: if the Sun was at an infinite distance, it would act equally upon them, and in the direction of parallel lines; their relative motion, therefore, would not be affected by an action which was common to both; but its distance, though very great compared with that of the Moon, cannot be considered as infinite: the Moon is alternately nearer and farther from the Sun than the Earth, and the straight line joining the centres of the Sun and Moon, forms angles more or less acute with the radius vector of the Earth. Thus the Sun

acts unequally and in different directions on the Earth and Moon; and from this diversity of action, inequalities must necessarily arise in the lunar motion, depending on the respective positions of the Moon and Sun. To determine these, we must at the same time consider the mutual actions and motions of these three bodies, the Sun, the Earth, and the Moon. This constitutes the famous problem of the three bodies, the exact solution of which surpasses the powers of analysis; but from the proximity of the Moon, compared with its distance from the Sun, and from the comparative smallness of its mass, an approximation may be obtained extremely near the truth. Nevertheless, the most delicate analysis is necessary to investigate all the terms, whose influence becomes sensible; of this the first steps that were made in this analysis afford sufficient proof.

Euler, Clairaut, and Dalembert, who resolved this problem nearly about the same time, agreed in finding by the theory of gravitation, the motion of the lunar peri-

gee only half as great as it appears to be from observation. From which Clairaut concluded that the law of attraction was not quite so simple as had been imagined; and he supposed it to consist of two parts, one varying inversely as the squares of the distances, and sensible only at the great distance of the planets from the Sun, and that the other, increasing in a greater ratio as the distance diminished, became sensible at the distance of the Moon from the Earth. This conclusion was vehemently opposed by Buffon: he maintained that since the primordial laws of nature should be the most simple possible, they could only depend on one *modulus*, and their expression, therefore, must consist of one single term. This consideration should no doubt lead us not to complicate the law of attraction, except in case of extreme necessity; at the same time our ignorance respecting the nature of this force, does not permit us to pronounce with certainty as to the simplicity of its expression. However this may be, the metaphysician was in the right this

time in his contest with the mathematician, who retracted his error on making this important discovery, that by carrying on the approximation farther than had been done at first, the law of attraction, reciprocally as the squares of the distances, gave the motion of the lunar perigee, exactly conformable to observation, which has since been confirmed by all those who have occupied themselves on this subject. It is impossible without the assistance of analysis, to explain the connection of all the inequalities of the Moon's motion with the combined action of the Sun and Earth upon this satellite. We shall observe, that the theory of universal gravitation has not only explained the motion of the node and of the perigee of the lunar orbit, together with the three great inequalities known by the names of *variation*, *evection*, and *annual equation*, all which astronomers had already recognized; but it has likewise developed a great number of others less considerable, which it would have been almost impossible to have found and ascer-

tained by observation alone. In proportion as this theory has been brought to perfection, have the lunar tables acquired additional precision. This satellite, once so refractory, deviates now but little from the tables: but to give them that degree of precision, which is yet wanting, will require investigations at least as extensive as those which have been already made; for in every case the first steps which lead to a discovery, and the last which bring it to perfection, are the most difficult. It is possible, nevertheless, without analysis to explain the cause of the annual variation of the Moon, and of its secular equation. I shall the more willingly stop to explain the causes of these equations, because it will be seen that from them are derived the greatest inequalities of the Moon, which the course of ages may develope to observers, but which at the present period have been almost insensible.

In its conjunctions with the Sun, the Moon is nearer to it than the Earth, and experiences from it a more considerable

action; the difference of the attractions o
the Sun upon these two bodies, tends t
diminish the gravity of the Moon toward
the Earth. In a similar manner in the op-
positions of the Moon to the Sun, this sa-
tellite being farther from the Sun than the
Earth, is more weakly attracted: thus the
difference of the actions of the Sun tends
also in this case to diminish the gravity of
the Moon to the Earth. In each case the
diminution is very nearly the same, and
equal to twice the product of the mass of
the Sun, by the quotient of the radius of
the lunar orbit, divided by the cube of the
distance of the Sun to the Earth. In the
quadratures, the action of the Sun upon the
Moon, decomposed in the direction of the
lunar orbit, tends to augment the gravity
of the Moon to the Earth: but this aug-
mentation is only half the value of the di-
minution which it experienced in the sysy-
gies. Thus from all the actions of the Sun
upon the Moon in the course of a synodical
revolution, there results a mean force in
the direction of the lunar radius vector,

which diminishes the gravity of this satellite, and it is equal to half of the product of the mass of the Sun, by the quotient of the radius of the lunar orbit, divided by the cube of the distance of the Sun from the Earth.

To find the ratio which this product bears to the gravity of the Moon, we may observe, that this force of gravity which retains it in its orbit is nearly equal to the sum of the masses of the Earth and Moon, divided by the square of their mutual distance; and the force which retains the Earth in its orbit is very nearly equal to the mass of the Sun divided by its distance from the Earth. According to the theory of central forces, explained in the second Book, these two forces are as the radii of the orbits of the Sun and of the Moon, divided respectively by the squares of the times of their revolutions. Hence it follows that the preceding product is to the gravity of the Moon, as the square of the time of the sidereal revolution of the Moon is to the square of the time of the sidereal revolution of the Earth. This

product therefore is very nearly $\frac{1}{179}$ of the lunar gravity, which by the mean action of the Moon is thus diminished by its $\frac{1}{358}$ part.

In consequence of this diminution, the Moon is sustained at a greater distance from the Earth, than if it was abandoned entirely to the action of its own force of gravity. The sector described by its radius vector is not altered, since the force which produces it is in the direction of this radius, but its real velocity and angular motion are diminished, and it is easy to see, that by placing the Moon at a greater distance, so that its centrifugal force might equal its gravity, diminished by the action of the Sun, and that its radius vector might describe the same sector that it would have described without this action; this radius would be augmented by its 358th part, and its angular motion diminished by a 179th part.

These quantities vary reciprocally as the cubes of the distances of the Sun to

nar orbit, but this orbit contracts again, as the Sun approaches its apogee; thus the Moon describes in space, a series of epicicloids whose centres are on the terrestrial orbit, and which dilate and contract as the Earth approaches or recedes from the Sun. From hence an inequality arises in the lunar motion, very similar to the equation of the centre of the Sun, with this difference that it retards this motion, when that of the Sun augments, and that it accelerates it when the motion of the Sun diminishes. These two equations are thus always affected with contrary signs. The angular motion of the Sun is, as we have shewn in the first Book, reciprocally as the square of its distance at the perigee; this distance being $\frac{1}{60}$th less than the mean distance, its angular velocity is augmented $\frac{1}{30}$th; the diminution of $\frac{1}{179}$th produced by the action of the Sun in the lunar motion, is then greater by a twentieth; the increase of this diminution is therefore the 3580*th* part of this motion. Hence it follows that the equation of the centre of the S---

is to the annual equation of the Moon, as a thirtieth of the solar motion is to the 3580th of the lunar motion, which gives * 2398″ for the annual equation. It is about a seventh part less according to observation, this difference depends on the quantities that have been neglected in this first calculation.

The secular equation of the Moon is produced by a similar cause with the annual equation. Halley first remarked this equation, which Dunthorn and Mayer have confirmed by a profound discussion of the observations. These two learned astronomers have proved that the mean motion of the Moon cannot be reconciled with modern observations, and with the eclipses observed by the Chaldeans and Arabians. They have attempted to represent them by adding to the mean longitudes of this satellite a quantity proportional to the square of the number of centuries elapsed before or after the year

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* 12′ 56″ 9.

1700. According to Dunthorn this quantity is * 30″9, for the first century: Meyer made it † 21″6, in his first tables which he increased to ‡ 27″8, in his last. And since that time Lalande, after a new investigation of the subject was led nearly to the same result as Dunthorn. The Arabian observations which have been chiefly made use of, are two eclipses of the Sun and one of the Moon, observed by Ibn Junis, near Cairo, towards the end of the tenth century, and extracted some time ago from a manuscript of this astronomer's existing in the library at Leyden. Doubts have risen concerning the reality of these eclipses; but the translation which M. Caussin has lately made of the part of this valuable manuscript which contains the observations has dissipated these doubts; it has moreover made us acquainted with twenty-five other eclipses observed by the Arabians, and which confirm the acceleration of the mean motion of the Moon.

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* 10″. † 7″. ‡ 9″.

Besides our modern observations compared with the Chaldean, are sufficient to establish the existence of the secular equation of the Moon. Delambre and Bouvard have determined, by means of a great number of observations both of the past and present century, the actual secular motion of this satellite, with a precision that leaves a very slight uncertainty: they found it only * 80″ less than that of Mayer, when the antient observations give a secular motion less by 6 or † 700 seconds. The lunar motion is therefore accelerated since the time of the Chaldeans, and the Arabian observations being made in the interval that separates them and confirming this supposition, it is impossible any longer to question the truth of it.

Now, what is the cause of this phenomenon? Does the theory of universal gravitation, which has so well explained the numerous inequalities of the Moon, account likewise for its secular variations? These questions are the more interesting to resolve, because if we succeed, we shall

obtain the law of these secular variations of the Moon, for it is evident that the hypothesis of an acceleration proportional to the time, as admitted by astronomers, is only approximative and cannot extend to an unlimited period.

This object has greatly occupied the attention of geometricians, but their researches for a long time fruitless, having discovered nothing either in the action of the Sun or planets on the Moon, nor in the figures not exactly spherical of this satellite and the Earth, that could change the mean motion of the Moon, some rejected the secular equation altogether, others to explain it, had recourse to different hypothesis, such as the actions of comets, the resistance of an ether, and the successive transmission of gravity. Yet the correspondence of the other celestial phenomena with the theory of gravitation is so perfect, that we could not observe without great regret, that the secular variation of the Moon appeared to refuse to submit to it, and continued the only exception to

a general and simple law whose discovery, by the grandeur and variety of the objects which it embraces, does so much honour to the human understanding. This reflection having determined me to reconsider this question after several attempts I was at last so fortunate as to discover its cause. *The secular equation of the Moon arises from the action of the Sun upon this satellite combined with the variation of the excentricity of the terrestrial orbit.* To form a just idea of this cause, we must recollect that the elements of the orbit of the Earth are subject to alteration from the action of the planets; its greater axis remains always the same, but its excentricity, its inclination to a fixed plane, and the position of its nodes and of its perihelion are incessantly changing. It must also be considered, that the action of the Sun upon the Moon diminishes by $\frac{1}{179}$, its angular velocity, and that this numerical co-efficient varies reciprocally as the cube of the distance of the Earth from the Sun. Now in expanding the inverse third power of

the distance, into a series arranged according to the sines and cosines of the mean motions of the Moon, and of their multiples, taking for unity the semi-major axis of the terrestrial orbit; it is found that this series contains a term equal to three times the half of the square of the excentricity of this orbit; the expression of the diminution of the angular velocity of the Moon, contains therefore a term equal to the 179th part of this velocity multiplied by three times half the square of this excentricity, or what is equivalent, equal to the product of this square, by the angular velocity of the Moon, divided by 119.33. If the excentricity of the terrestrial orbit was constant, this term would be confounded with the mean angular velocity of the Moon; but its variation though very small, has nevertheless in progress of time a sensible influence on the motion of the Moon. It is evident that this motion will be accelerated when the excentricity diminishes, which has been the case ever since the most ancient observations to the

sent time, this acceleration will be changed into a retardation, when the excentricity arrived at its *minimum* will cease to decrease and begin to augment.

In the interval from 1700 to 1800, the square of the excentricity of the terrestrial orbit diminishes 0.000001532b, half the greater axis being taken as unity, the corresponding increase in the angular velocity, of the moon is therefore 0.000000012 8425 of this velocity : this increase taking place successively and proportional to the time, its effect on the Moon's motion is only half what it would be if in the whole course of the century it was the same as in the end. To determine therefore this effect or the secular equation of the Moon at the end of a century, reckoning from 1700, we must multiply the secular motion of the Moon by the half of the very small increase in its angular velocity, but in a century the motion of the Moon is *534740545.4, which gives †34″ 337 for its secular equation.

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* 481266° 29′ 27″ † 11″ 1.

As long as the diminution of the square of the excentricity of the terrestrial orbit may be supposed proportional to the time, the secular equation of the Moon will increase sensibly as the square of the times; it would be sufficient therefore to multiply *34″337* by the square of the number of centuries contained between 1700 and the time for which the calculation is made. But I have found that in going back to the observations of the Chaldeans, the term proportional to the cube of the times, in the expression in series, of the secular equation of the Moon, becomes sensible, this term is equal to ‡0″13574 for the first century; it should be multiplied by the cube of the number of centuries reckoned from 1700, the product being taken as negative for the centuries anterior to this epoch. The mean action of the Sun upon the Moon depends also on the inclination of the lunar orbit to the ecliptic, and we

‡ 0″ 122165.

might suppose that the position of the ecliptic being variable, there should result inequalities in the motion of this satellite similar to those produced by the diminution of the excentricity of the terrestrial orbit; but the lunar orbit is constantly brought back by the action of the Sun to the same inclination to that of the Earth, so that the greatest and least declinations of the Moon are, in consequence of the secular variation in the obliquity of the ecliptic, subject to the same changes as the declinations of the Sun.

This constancy in the inclination of the lunar orbit is confirmed by all observations both ancient and modern.

The excentricity of this orbit experiences equally only an insensible alteration from the change of the excentricity of the terrestrial orbit.

It is not thus with the variation of the motion of the nodes and perigee, to which attention must be paid indispensibly, in investigations, the object of which are to perfect the lunar tables. In submitting

these variations to analysis, I have found that the influence of the terms depending on the square of the perturbating force, and which, as we have seen, double the mean motion of the perigee, is yet greater on the variation of this motion. The result of this intricate analysis has given me a secular equation to be subtracted from the mean motion of the perigee, and equal to thirty-three tenths of the secular equation of the lunar motion; so that the mean motion of the perigee is retarded, when that of the Moon accelerates. I have found likewise in the motion of the nodes of the lunar orbit upon the true ecliptic, a secular equation to be added to their mean longitude, and equal to seven tenths of the secular equation of the mean motion. Thus the motion of the nodes is retarded like that of the perigee when that of the Moon augments, and the secular equations of these three motions, are constantly in the proportion of the numbers 7, 33, and 10.

Future ages will develope these great inequalities which will produce one day variations at least equal to a fortieth of the circumference, in the secular motion of the Moon, and to a twelfth of the circumference in that of its perigee. These inequalities do not always continue increasing, they are periodical like those of the excentricity of the terrestrial orbit on which they depend, and do not re-establish themselves till after millions of years.

They must at length alter the periods which have been devised for the purpose of comprehending the entire numbers of revolutions of the Moon relatively to it nodes, to its perigee, and to the Sun, periods which differ sensibly in various parts of the immense period of the secular equation.

The luni-solar period of six hundred years, has been rigorously exact at a certain period which it would be easy to find by analysis if the masses of the planets were well determined; but this determination, so desirable for the perfection of astrono-

nomical theory, is yet wanting. Fortunately Jupiter, whose mass we know exactly, is the planet which has the greatest influence on the secular equation on the Moon.

Already ancient observations, notwithstanding their imperfection, confirm these inequalities, and we may trace their progress either in these ancient observations or in the astronomical tables which have succeeded them to the present time. We have seen that the ancient eclipses, had made known the acceleration of the Moon's motion, before the theory of gravity had developed the cause.

In comparing modern observations and the eclipses observed by the Arabians, Greeks, and Chaldeans, with this theory, we find an agreement between them that appears surprising, when we consider the imperfection of ancient observations, the vague manner in which they have been transmitted to us, and the uncertainty which still exists concerning the excentricity of the earth's orbit, from our doubts re-

specting the masses of Venus and Mars. The developement of the secular equations of the moon is one of the most proper data to determine these masses.

It was particularly interesting to verify the theory of gravity, relatively to the secular equations of the motion of the Moon's nodes and perigee, the knowledge of which we owe to it. Astronomers not having attended to these equations, in the comparison of ancient and modern observations, should have found these motions too rapid; while at the same time they assigned too small a mean motion to the Moon when they did not consider its secular equation. It is this which Bouvard has confirmed by the comparison of a great number of modern observations. Above five hundred observations by de la Hire, Flamsteed, Bradley, and Maskelyne, disposed in the most favourable manner, and carefully discussed, have informed him that the secular motion of the perigee in the lunar tables inserted in the third edition of de Lalande's astronomy,

must be diminished by about * fifteen minutes and three quarters. This motion thus erected no longer represents the ancient eclipses, which from hence demonstrates the existence of the secular equation of the perigee of the Moon.

To discover if the magnitude of this equation is the same as is given by the law of universal gravitation, Bouvard has first compared with the tables above-mentioned, twenty-one eclipses observed by the Greeks and Chaldeans, and this comparison has given him very nearly the secular equation of the perigee equal to thirty-three tenths of that of the mean motion: thereby two eclipses observed by the Arabians have conducted him to the same result, which he has again discovered by sixty eclipses, observed since the revival of astronomy in Europe till the commencement of the present century. This remarkable agreement between the results drawn from observations made at such very different epochs does not leave any doubt concerning the existence and magnitude of

* 3′ 30″.

the secular equation of the lunar perigee, and confirms in an incontestible manner the relation of thirty-three to ten, which the theory of gravity establishes between this equation, and that of the Moon's mean motion. Bouvard has also confirmed by the comparison of the same eclipses, the secular equation of the nodes; and he has found that their motion in a century, given in the tables already cited ought to be diminished by *537″

The mean motions and the epochs of the tables of the Almageste and of the Arabians, indicate evidently these three secular equations of the lunar motion. The tables of Ptolemy are the result of immense calculations made by this astronomer and by Hipparchus; the labour of Hipparchus has not descended to us: we only know from the evidence of Ptolemy, that he had taken the greatest care to choose eclipses the most advantageous to the determination of the elements of which they were in

* 2′ 50″ 2.

search. Ptolemy after two centuries and a half of new observations found nothing to change in the mean motion of the Moon as determined by Hipparchus; he corrected the motion of the nodes and perigee but a very small quantity; there is therefore reason to believe that the elements of the lunar tables of Ptolemy have been determined by a great number of eclipses, of which he only preserved those that appeared to him to coincide most with the mean results which had been obtained by Hipparchus and himself. Eclipses only make known correctly the mean sinodical motion of the moon, and its distances from its nodes and its perigee: we can only then depend upon these elements in the tables of the Almageste; now in going back to the first epoch of these tables, by means of motions determined only by modern observations, we do not find the mean distances of the Moon from its nodes, its perigee, and from the Sun, that are given in these tables at this epoch. The quantities which must be added to these distances, are very nearly

these which result from the secular equations; the elements of these tables confirm therefore at the same time, the existence of these equations and the values which I have assigned to them.

The motions of the Moon relative to its nodes, to its perigee, and to the Sun, being slower in the tables of the Almageste than in our days, indicate also in these three motions an acceleration equally indicated whether by the corrections that Albategnius eight centuries after Ptolemy, made to the elements of these tables, by comparing them with a great number of eclipses observed in his time; or by the epochs of the tables which Ibn Junis constructed about the year one thousand, from the assemblage of the Chaldean, Greek and Arabian observations.

It is remarkable that the diminution of the excentricity of the terrestrial orbit should be much more sensible, in the lunar motion than in itself. This diminution which, since the most ancient eclipse we are acquainted with, has not altered

the equation of the suns centre * 15′, has produced a † 100″ of variation in the Moon's longitude, and nearly ‡ 9° variation in its mean anomaly; we could hardly suspect it from the observations of Hipparchus and Ptolemy. Those of the Arabians indicated it with much probability; but the ancient eclipses, compared with the theory of gravitation, leave no doubt on this subject.

Here we see an example of the manner in which phenomena as they are developed, lead us to the knowledge of their true causes. When only the acceleration of the mean motion of the Moon was known, it might be attributed to the resistance of ether, or to the successive transmission of gravity. But analysis proves that these two causes cannot produce any sensible alteration in the mean motion of the nodes and of the lunar perigee, and that alone would suffice to exclude them, even when the true cause of the variations observed in these motions was unknown.

* 8′ 6″ † 90″ ‡ 8° 6′.

The agreement of theory with observation proves that if the mean motions of the Moon are altered by causes foreign to the principle of universal gravitation, their influence is very small, and hitherto insensible.

Some partizans of final causes have imagined that the Moon was given to the Earth to afford it light in the absence of the Sun. But in this case nature would not have attained the end proposed, since we are often deprived at the same time of the light of each of them. To have accomplished this end it would have been sufficient to have placed the Moon at first in opposition to the Sun and in the plane of the ecliptic, at a distance from the Earth equal to one hundredth part of the distance of the Earth from the Sun, and to have given to the Earth and to the Moon, velocities parallel and proportional to their distances from the Sun. In this case the Moon being constantly in opposition to the Sun, would have described round it an ellipse similar to that of the Earth,

these two bodies would thus have succeeded each other above the horizon, and as at this distance the Moon would not be eclipsed, its light would always replace that of the Sun.

Other philosophers, struck with the singular opinion of the Arcadians who thought themselves more ancient than the Moon, have imagined that this satellite may formerly have been a comet which passing near the Earth may have been forced by its attraction to accompany it. But by reascending by analysis back into the most distant ages, we find that the Moon has always moved in an orbit nearly circular, in the same manner as the planets round the Sun. Neither, therefore, has the Moon nor any other satellite ever been a comet.

CHAP. VI.

Of the Perturbations of the Satellites of Jupiter.

The first inequalities which observation discovered in the motion of these bodies, are also the first which are derived from the theory of their mutual attractions. We have seen in the second Book, that there exists

1. An equation in the motion of the first satellite equal to * 5258″, multiplied by the sine of double the excess of the mean longitude of the first satellite above that of the second.

2. An equation in the motion of the second satellite equal to † 11923″, multiplied by the sine of the excess of the first satellite above that of the third.

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* 28′ 23″ 5. † 1° 4′ 23″.

3. An equation in the motion of the third satellite equal to * 827″, multiplied by the sine of the excess of the longitude of the second satellite above that of the third.

Not only the theory of gravity gives these inequalities, as Lagrange and Bailly were the first to remark, but it shews us also, what observation seemed to indicate, that the inequality of the second satellite is the result of two inequalities, of which one being caused by the action of the first satellite, varies as the sine of the excess of the longitude of the first satellite above that of the second; and the other, produced by the action of the third, varies as the sine of double the excess of the longitude of the second satellite above that of the third. Thus the second satellite experiences a perturbation from the action of the first, similar to that which itself causes in the third; and it experiences from the third a similar perturbation to that which itself causes in the first.

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* 1′ 27″ 9.

These two inequalities are combined into one in consequence of the relation which exists between the mean motions and the mean longitudes of the three first satellites, for the mean motion of the first satellite *plus* twice that of the third, is equal to three times that of the second; and the mean longitude of the first satellite *minus* three times that of the second *plus* twice that of the third is constantly equal to a semi-circumference: but will these relations always exist, or are they only approximative, and will not the two inequalities of the second satellite, at present combined, be separated in the course of time? It is to theory that we must apply for a solution to this question.

The approximation which the tables gave to the preceding relations, made me suppose that they were rigorously exact; for it was against all probability that chance should have originally placed the three first satellites at the precise distances and positions suitable to the above relation: it was therefore extremely probable that it arose

from some particular cause; I looked therefore for this cause in the mutual action of the satellites. A scrupulous investigation of this action, has shewn me that it has caused these relations to be rigorously exact; from whence I concluded, that in determining again by the examination of a great many distant observations, the mean longitudes of the three first satellites, it would be found that they would approximate still more to these relations, to which the tables should be made exactly to agree. I had the satisfaction of seeing this consequence of the theory confirmed, with remarkable precision, by the researches which Delambre has lately made concerning the satellites of Jupiter. It is not necessary that these relations should have taken place exactly at their origin, it was enough that they did not greatly differ, then the mutual actions of the satellites upon each other were sufficient to subject them to this law, and to maintain it unaltered; but the little difference between this and the primitive relation, has given rise to a small inequality of

an arbitrary extent, and unequally distributed *among the three satellites, and which* I have distinguished under the name of *libration.* The two constant arbitrary quantities of this inequality, replace whatever arbitrary quantity is made to disappear by the two preceding relations, in the mean motions and in the epochs of the mean longitudes *of the three first satellites; for the number* of arbitrary quantities included in the theory of a system of bodies is necessarily sextuple the number of bodies: as observation does not indicate this inequality, it must evidently be very small, and even insensible.

The *preceding relations would still* subsist, even if the mean motions of the satellites were subject to secular variations analogous to that in the motion of the Moon. They would subsist also in the case of these motions being altered by the resistance of a medium, or by other causes, provided their *effects were so small as not* to be perceived in less than a century. In all these cases the secular equations so arrange themselves, by the reciprocal action

of the satellites, that the secular equation of the first *plus* twice that of the third, will be constantly equal to three times that of the second. Thus the three first satellites of Jupiter form a system of bodies connected together by the preceding relations and inequalities, which their mutual action will maintain for ever, except some external cause should abruptly derange their respective positions.

The theory of gravitation has also enabled me to ascertain the cause of the singular variations observed in the excentricity of the orbit of the third satellite, which I mentioned in the second Book. These variations depend on two equations of the centre, very distinct from each other, to which its motion is subject, of which one relates to the perijoves proper to this satellite, and the other to the perijove of the fourth. The excentricities of the orbits of the four satellites, and their perijoves, are connected with each other by the mutual action of these bodies, in consequence of which the excentricity of the fourth sa-

tellite extends itself over the three others, but more feebly as they are more remote. It is very sensible in the orbit of the third, and combining itself with the excentricity peculiar to this orbit, it produces in the motion of the third satellite a compound equation of the centre, whose greatest value incessantly varies, and depends on a perijove, the motion of which is not uniform. The longitude of the perijove of the fourth satellite was * 159° 48, at the commencement of 1700, and its annual and sidereal motion is † 7852″; the longitude of the perijove proper to the third satellite was ‡ 194° 11 at the same period, and its annual and sidereal motion is ‖ 29776″. These perijoves coincided in 1684, and the two equations of the centre formed a single one, equal to their sum, the greatest value of which amounted to § 2661″. In 1775, these perijoves having arrived at contrary positions, the two equations of their centres

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133° 29′ 13″ 2. † 42′ 14″ 4. ‡ 174° 41′ 56″ 4.
‖ 2° 40′ 47″ 4. § 14′ 22″ 1.

formed one equal to the difference, whose value was only * 759″. This explains the reason why Wargentin found, by comparing the observations, that the excentricity of this satellite was the greatest towards the beginning of the century, and least about the year 1760. He had at first endeavoured to explain these variations, by means of two equations of the centre, but not being aware that one of them depended on the perijove of the fourth satellite, and having also assigned to them incorrect values, he was forced to abandon them, and to recur to the hypothesis of a variable excentricity, whose variations he determined by experiment.

The mutual action of the satellites of Jupiter, produces at every instant a variation in the positions of their orbits. This is what the theory, compared with the observations, gives upon this subject. The equator of Jupiter is inclined † 34444″ to the plane of the orbit of that planet, the

* 4′ 5″ 9. † 3° 51′ 59″ 8.

longitude of its ascending node was * 347° 8519; at the commencement of 1700, its annual and sidereal motion is about † 6″. The orbit of the first satellite is only inclined ‡ 22″ to the plane of the equator of Jupiter; its nodes on this plane, coincide with the nodes of the same plane with the orbit of Jupiter, the orbit of the satellite being between these two planes.

The orbit of the second satellite moves in a fixed plane, inclined ‖ 221″ to the equator of Jupiter, and which passes through the line of the nodes of this equator, between this last plane, and that of the orbit of Jupiter. The orbit of Jupiter is inclined § 5182″ to this fixed plane, and its nodes with this plane have a retrograde motion, whose annual and sidereal value is equal ¶ 13° 3488, and whose period is thirty Julian years. The longitude of the ascending node was ** 179° 5185, in 1700.

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* 313° 4′. † 1″ 9. ‡ 7″ 1. ‖ 1′ 11″ 6. § 27′ 58″ 9. ¶ 12° 0′ 50″ 1. ** 161° 33′ 59″ 9.

The orbit of the third satellite moves on a fixed plane, inclined * 1030″ to the equator of Jupiter, and which passes through the line of the node of this equator, between this last plane, and that of the orbit of Jupiter; the orbit of the satellite is inclined † 2244″ to this fixed plane, and its nodes with this plane have a retrograde motion, whose annual and sidereal value is ‡ 2° 9149, and period 137 years; the longitude of its ascending node was § 136° 9630 in 1700. Astronomers who had recognised the motion of this node by observations, supposed the orbits of the second and third satellites in motion on the equator itself of Jupiter, but they were obliged by the observations to diminish a little the inclination of this equator, with the orbit of Jupiter, when they considered the motion of the third satellite.

Lastly, the orbit of the fourth satellite moves in a fixed plane, inclined ¶ 4630″ to

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* 5′ 33″ 7. † 12′ 7″. ‡ 2° 37′ 24″ 2.
§ 123° 16′. ¶ 25′.

the equator of Jupiter, and which passes through the line of nodes of this equator, between this last plane and that of the orbit of Jupiter. The orbit of the satellite is inclined * 2772″ to this fixed plane, and its nodes with it have a retrograde motion, whose annual and sidereal value is † 7519″, and period 532 years, the longitude of the ascending node is ‡ 153°5185 in 1700.

The inclination of the orbit of the fourth satellite, with the orbit of Jupiter, continually varies in consequence of this motion. Having arrived at its mininum, about the end of the last century, it remained nearly stationary for a great number of years, and the nodes of the orbit of the satellite, with that of Jupiter, have had a direct annual motion of about § 8′. This circumstance was recognised by observations, and astronomers availed themselves of it, in their tables of this satellite; but for several years back,

* 14′ 58″. † 40′ 36″. ‡ 138° 10′.
§ 4′ 19″ 2.

observation has indicated a very sensible augmentation in the inclination of its orbit with that of Jupiter, which without the assistance of the theory, would have rendered the construction of the tables very difficult.

It is satisfactory to the geometrician to see these singular phenomena arise from analysis, which are perceptible to observation, but being at the same time the result of several simple inequalities, are too complicated for astronomers to ascertain their laws.

The different planes which we have just described, and on which the orbits of the satellites move, are not rigorously fixed; the plane of the equator of Jupiter, carrying them on in his motion, so that their nodes, with the orbit of this planet, being constantly the same with those of its equator; their inclinations, with the plane of this orbit, are always proportional to that of the equator. But all these motions are insensible, from the time of the discovery of the satellites, to the present day.

The orbit of each satellite participates a little in the motion of the adjoining orbit; for every thing is connected, in a system of bodies mutually subject to the action of each other. The satellites form round Jupiter a system similar to that of the planets round the Sun; and as their revolutions are very rapid, they present us in the short space of time since their discovery, all the great changes which a series of ages will produce in the planetary system. Thus the agreement of the theory of gravitation, with the variations observed in the motions of the satellites, leaves us no reason to doubt the variations which it indicates in the orbits of the planets, and which the most ancient observations, compared with our own, would scarcely render sensible.

This theory has banished all empiricism from the tables of the satellites of Jupiter. Those which Delambre has lately published, borrowing only from observation such data as necessarily depend on it, have the advantage of extending to all ages by

rectifying these data as they become better known. We may conceive, that to establish the theory which served for a basis to these tables, it was necessary to know both the masses of the satellites, and the compression of Jupiter.

Five data, derived from observation, are necessary to determine these five unknown quantities. Those which I employ are the two principal inequalities of the first and second satellites; the period of the variations of the inclination of the orbit of the second satellite; the equation of the centre of the third satellite, which depends on the perijove of the fourth. Finally, the motion of this perijove. Taking for unity the mass of Jupiter, the masses of its satellites, as deduced from the preceding data, are as follows:

I. Satellite . .	0.0000172011
II. Satellite . .	0.0000237103
III. Satellite . .	0.0000872128
IV. Satellite . .	0.0000544681

These values may be corrected, when in the progress of time we become better ac-

quainted with the secular variations of the orbits of the satellites.

The ratio of the two axes of Jupiter, resulting from these data, is equal 0.93041. This quantity has been measured several times, and the mean result is $\frac{13}{14}$, or 0.929, which does not differ a sensible quantity from the preceding result. But considering the great influence which the compression of the figure of Jupiter, has on the motion of the nodes, and of the perijoves of the satellites, we perceive that the ratio of the two axes of Jupiter, is given with greater precision by the observations of the eclipses, than by the most exact measures taken with a micrometer. The agreement of these measures, with the result of the theory, proves in a most satisfactory manner, that the action of gravity towards Jupiter, is composed of the gravities towards each of its particles; since in reasoning from this principle, we find the compression such as it really appears to be.

The eclipses of the first satellite of Ju-

piter, gave rise to the discovery of the successive motion of light, which the phenomenon of aberration has ascertained with still greater precision. It appeared to me that the theory of the motion of this satellite being now better known, and the observation of its eclipses become more numerous, their discussion should give the quantity of aberration more exactly than by direct observation. Delambre undertook this investigation at my request, and found the entire quantity of aberration * 62″5, which is exactly that which Dr. Bradley derived from his observations. It is very curious to observe such a perfect agreement, in results which have been obtained by such very different methods.

It follows from this agreement, that the *velocity of light is uniform* through the whole space comprehended by the terrestrial orbit. In fact, the velocity of light given by the aberration, is that which sub-

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* 20″ 2.

sists at the circumference of the terrestria orbit, and which, being combined witl the motion of the Earth, produces thi phenomenon. The velocity of light, a given by the eclipses of the satellites o Jupiter, is determined by the time whicl light employs to traverse the terrestria orbit; these two velocities being the same the velocity is uniform through the whol length of the diameter of the terrestria orbit. It results also from these eclipses that the velocity of light is uniform througl the whole diameter of the orbit of Jupi-ter; for, from the excentricity of this or-bit, the effect of the variations in the radi vectores, is very sensible in the eclipses o the satellites; and these exactly corres-pond to the hypothesis of a uniform velo-city, in the motion of light.

If light is an emanation from luminou bodies, the uniformity of its velocity re-quires that it should be projected fro each of them with the same force, an that its motion should not be sensibly re-tarded by their attraction. If we suppos

light to consist in the vibrations of an elastic fluid, we must then, to explain the uniformity of their velocity, suppose the density of the fluid throughout the whole extent of the planetary system, proportionate to its elasticity. But the simplicity with which the aberration of the stars, and the phenomena of the refraction of light, in passing from one medium to another, are explained by considering light as an emanation from a luminous body, renders this hypothesis extremely probable.

CHAP. VII.

Of the Figure of the Earth and Planets, and of the Law of Gravity at their Surface.

It has been shewn in the First Book, what we have learnt from observations on the figure of the Earth, and of the planets: let us compare these results with those of universal gravitation.

The force of gravity towards the planets, is composed of the attractions of all their particles. If their mass was fluid, and without motion, their strata would be spherical, those nearer the centre being more dense. The force of gravity at their exterior surface, and at any distance whatever, without the sphere, would be exactly the same, as if the whole mass of the planet was compressed into the centre of gravity. It is in consequence of this remarkable property, that the Sun, the

planets, comets, and satellites, act upon each other, very nearly as if they were so many material points. At very great distances. the attraction of the particles of a body of any figure, which are the most remote, and those which are nearest the particle attracted, compensate each other in nearly the same manner as if they were united in the centre of gravity; and if the ratio of the dimensions of the body be considered as a very small quantity of the first order, this result will be exact to a quantity of the second order. But in a sphere, it is rigorously true, and in a spheroid differing but little from a sphere, it is of the same order as the product of its excentricity, by the square of the ratio of its radius, to the distance of the point attracted. This property of the sphere, of attracting as if its mass was concentrated in its centre, contributes greatly to the simplicity of the motions of the heavenly bodies. It does not belong exclusively to the law of nature, it equally appertains to the law of the attraction

varying proportionably to the simple dis tance, and cannot belong to any other lav but those formed by the addition of thes two. And of all the laws which rende the force of gravity nothing at an infinit distance, that of nature is the only on in which the sphere possesses this pro perty.

According to this law, a body place within a spherical stratum of uniforn thickness, is equally attracted by all it parts, so as to remain at rest in the mids of the various attractions which act upo it. The same circumstance takes place i an elliptic stratum, when the exterior an interior surfaces are similar and similarl situated. Supposing therefore the planet to be spheres of homogeneous matter, th force of gravity in their interior, must di minish as the distance from the centre; fo the exterior part, relatively to the attract ed particle, contributes nothing to its gra vity, which entirely consists of the attrac tion of the internal sphere, whose radiu is equal to the distance of this point fro

the centre. But this attraction is equal to the mass of the sphere, divided by the square of the radius, and the mass, is as the cube of this same radius. The force of gravity on the attracted particle, is therefore proportional to the radius. But if, (as is probably the case) the strata are more dense as they approach the centre, the force of gravity will diminish in a less ratio, than in the case of homogenity. The rotary motion of the planets causes them to differ a little from the spherical figure. The centrifugal force arising from this motion, causing the particles situated at the equator, to recede from the centre, and produce a flattening of the poles.

Let us consider first the effects of this circumstance in the most simple case, of the Earth's being an homogeneous fluid, and the whole force of gravity residing in its centre, and varying reciprocally as the square of the distance from this point. It will then be easy to prove that the terrestrial spheroid is an ellipsoid of revolution; for if we conceive two columns of fluids,

communicating with each other at the centre, terminating, the one at the pole, the other at any point in the surface, these two columns ought to be in equilibrio. The centrifugal force does not alter the weight of the column directed to the pole, but diminishes the weight of the other column. This force is nothing at the centre of the Earth, and at the surface is proportional to the radius of the terrestrial parallel, or very nearly, as the cosine of the latitude; but the whole of this force is not entirely employed in diminishing the force of gravity; for these two forces making an angle with each other, equal to the latitude, the centrifugal force, decomposed according to the direction of gravity, is weakened in the ratio of the cosine of this angle to radius. Thus, at the surface of the Earth, the centrifugal force diminishes the force of gravity, by the product of the centrifugal force at the equator, by the square of the cosine of the latitude; therefore the mean value of this diminution in the length of a fluid co-

lumn, is the half of this product, and since the centrifugal force is $\frac{1}{289}$ of the force of gravity at the equator, this value is $\frac{1}{578}$ of the force of gravity, multiplied by the square of the cosine of the latitude. And since it is necessary, for the maintenance of the equilibrium, that the column by its length shall compensate the diminution of its weight, it should surpass the polar column by $\frac{1}{578}$ of its length, multiplied by the square of the above cosine. Thus the augmentation of the radii, from the pole to the equator, is proportional to the squares of these cosines, from whence it is easy to conclude, that the Earth is an ellipsoid of revolution, the equatorial and polar axis of which were in the proportion of 578 to 577.

It is evident that the equilibrium of the fluid mass would still subsist, supposing that a part should consolidate itself in the interior, provided the force of gravity remains the same.

To determine the law of gravity at the surface of the Earth, we should

observe that the force of gravity to any point on this surface, is less than that at the pole, from its being situated farther from the centre. This diminution is nearly equal to double the augmentation of the terrestrial radius; it is equal therefore to the product of the $\frac{1}{289}$ part of the force of gravity by the square of the cosine of the latitude. The centrifugal force diminishes likewise the force of gravity by the same quantity; thus by the union of these two causes, the diminution of gravity from the pole to the equator, is $= 0{,}00694$, multiplied by the square of the cosine of the latitude, the force of gravity at the equator being taken as unity.

It has been shewn in the First Book, that the measures of meridional degrees, give the Earth an ellipticity greater than $\frac{1}{578}$, and that the measures of the pendulum indicate a diminution in the force of gravity, from the poles *to the equator*, less than 0.00694, and equal to 0.00567. The measures of the degrees and of the pendulum concur, therefore, to prove that

the force of gravity is not directed to a single point, but is composed of the attractions of all the particles of the Earth.

This being the case, the law of gravity depends on the figure of the terrestrial spheroid, which depends itself on the law of gravity. It is this mutual dependance of the two unknown quantities on each other, that renders the investigation of the figure of the Earth very difficult. But fortunately the elliptic figure, the most simple of all the re-entering figures next to the sphere, satisfies the condition of the equilibrium of a fluid mass, subject to a motion of rotation, and of which all the particles attract each other reciprocally, as the squares of the distance. Newton, upon this hypothesis, and supposing the Earth a homogeneous fluid, found the ratio of the equatorial to the polar axis, to be 230 to 229.

It is easy to determine the law of variation of the force of gravity on the Earth, upon this hypothesis. For this purpose, let us consider two different points

situated on the same radius, drawn from the centre to the surface of an homogeneous fluid, in equilibrio. All the similar elliptic strata, which cover any one amongst them, contribute nothing to its gravity. The resulting force of all the attractions which act on it, is derived entirely from the attraction of the interior spheroid, similar to the entire spheroid, and whose surface passes through the point in question. The similar and similarly situated particles of these two spheroids, attract the interior point, and the corresponding point of the exterior surface, proportionally to their masses, divided by the squares of their distances. These masses are in the two spheroids, as the cubes of their similar dimensions, and the squares of their distances, are as the squares of these dimensions. The attractions on similar particles, are proportional therefore to these dimensions; from which it follows, that the entire attractions of the two spheroids, are in the same ratio, and their directions parallel.

The centrifugal forces of the two points, now under consideration, are likewise proportional to the same dimensions. Therefore the force of gravity in each of them being the result of these two forces, will likewise be proportional to their distances from the centre of the fluid mass.

Now, if we conceive two fluid columns directed as before, to the centre of the spheroid, one from the pole, and the other from any point on the surface, it is evident, that if the ellipticity of the spheroid is very small, that is, if it differs but little from a sphere, that the force of gravity, decomposed according to the directions of these columns, will be nearly the same as the total gravity. Dividing, therefore, the length of these columns into an equal number of parts, infinitely small and proportional to their lengths, the weights of the corresponding parts will be to each other as the products of the lengths of the columns, by the force of gravity at the points of the surface where they terminate. The whole weight of these columns will there-

fore be to each other, in this ratio; and as these weights must be equal, to be in equilibrio, the force of gravity at their surface must consequently be reciprocally, as the length of these columns. Thus the length of the radius of the equator, surpassing the radius at the pole a 230th part, the force of gravity at the pole should likewise exceed that at the equator a 230th part.

This supposes the elliptic figure sufficient for the equilibrium of a homogeneous fluid mass. Maclaurin has demonstrated this in a beautiful manner, from which it results, that the equilibrium is rigorously possible; and that, if the ellipsoid differs little from a sphere, the ellipticity will be equal $\frac{5}{4}$ of the quantity, which expresses the proportion of the centrifugal force, to that of gravity, under the equator.

Two different figures of equilibrium may correspond to the same motion of rotation. But the equilibrium cannot exist with every motion of rotation. The short-

est period of rotation of an homogeneous fluid in equilibrio, of the same density as the Earth, is * 0.1009 of a day, and this limit varies reciprocally, as the square root of the density. When the motion of rotation increases in rapidity, the fluid mass becoming more flattened at the poles, its period of rotation becomes less, and ultimately falls within the appropriate limits of a state of equilibrium. After a great many oscillations, the fluid, in consequence of the friction and resistances which it experiences, fixes itself at last in that state which is *unique*, and determined by the primitive motion of rotation. The axis drawn through the centre of gravity, of the fluid mass, and relative to which the moment of the forces was a maximum, at the origin, becomes the axis of rotation.

The preceding results afford us an easy method of verifying the hypothesis of the homogenity of the Earth. The irregularity of the measured degrees, may be

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* $2^h\ 25'\ 17''$.

supposed to leave too much uncertainty, as to the ellipticity, to enable us to decide, if it is really such as the above hypothesis requires. But the regular increase of the force of gravity, from the equator to the pole, as determined by experiments on the pendulum, is sufficient to throw great light upon the subject.

In taking as unity the force of gravity at the equator, its increase at the pole, according to the hypothesis of homogenity, should be equal $\frac{1}{230} = 0.00435$. But by observations on the pendulum, this increase is 0.00567 : the Earth therefore is not homogeneous. And indeed it is natural to suppose, that the density of the strata increase as they approach the centre. It is even necessary, for the stability of the equilibrium of the waters of the ocean, that their density should be less than the mean density of the Earth ; otherwise, when agitated by the winds and other causes, they would overflow their limits, and inundate the adjoining continents.

The homogenity of the Earth being thus excluded by observation, we must, to

determine its figure, suppose the sea covering a nucleus, composed of different strata, diminishing in density from the centre to the surface. Clairaut has demonstrated, in his beautiful work, that the equilibrium is still possible, in the supposition of an elliptic figure at the surface, and of the strata of the interior nucleus. In the most probable hypothesis, relative to the law of the densities and ellipticities of these strata, the ellipticity of the Earth is less than in the case of homogenity, and greater than if the force of gravity was directed to a single central point. The increase of the force is greater than in the first case, and less than in the second. But there exists between the increase of the force of gravity, taken as unity at the equator, and the ellipticity of the Earth, this remarkable analogy, that in all the hypotheses relative to the structure of the internal nucleus, which the sea incloses, the ellipticity of the Earth is just so much less than that which would take place in the case of homogenity, as

the augmentation of the force of gravity exceeds that which should exist, according to the same supposition, and reciprocally, so that the fractions expressing the ellipticity, and the augmentation of the force of gravity always together, make a constant quantity equal $\frac{5}{2}$, of the proportion of the centrifugal force, to the force of gravity at the equator, which, on the Earth is $\frac{1}{115.2}$.

In attributing an elliptic figure to the strata of the terrestrial spheroid, the increase of its radii, the increase of the force of gravity, and the diminution of the degrees, from the pole to the equator, will vary as the squares of the cosine of the latitude, and these are connected with the ellipticity of the Earth, in such a manner, that the total increase of the radii is equal to the ellipticity. The total diminution of the degree, is equal to the ellipticity, multiplied by three times the degree at the equator; and the total increase of the force of gravity, is equal to the force of gravity at the equator, multiplied by the excess of $\frac{1}{115.2}$, above the ellipticity.

Thus the ellipticity of the Earth may be determined, either by direct measurement of degrees, or by observations on the length of the pendulum.

The observations of the pendulum give 0.00567, for the increase of the force of gravity, which taken from $\frac{1}{115,2}$, gives $\frac{1}{332}$, for the ellipticity of the Earth. If this hypothesis of the ellipse be conformable to nature, it should agree with the measures of degrees; but it implies errors that are altogether improbable: and this circumstance, joined to the difficulty of reconciling all these measures to the same elliptic figure, proves that the figure of the earth is much more complicated than had been believed. This will not appear surprising, if we consider the different depths of the sea, the elevation of the continents, and islands above its level, the heights of mountains, and the unequal density of the water, and different substances which are at the surface of this planet.

To embrace, in the most general manner

possible, the theory of the figure of the Earth and planets, it is necessary to determine the attraction of spheroids, differing little from spheres, and formed of strata, variable both in figure and density, according to any law whatever.

It will remain then to determine the figure which will agree with the equilibrium of a fluid, expanded over its surface, for we must imagine the planets covered with a fluid similar to the Earth, or their form would be entirely arbitrary. Dalembert has given, for this purpose, an ingenious method, which extends to a great number of cases, but which is deficient in that simplicity so desirable in such complicated investigations, and which constitutes their principal merit.

A remarkable equation of partial differences relative to the attraction of spheroids, led me, without the aid of integrations, and by differential methods only, to general expressions, for the radii of the spheroids; for the attractions upon any points whatever, either within the sur-

faces, or without them; for the condition of equilibrium of the fluids that surround them; for the law of gravity, and for the variation of the degrees at the surface.

All these quantities are connected with each other, by analogies extremely simple, from which results an easy method of verifying all the hypotheses that may be formed to represent either the variation of the force of gravity, or that of the values of different degrees of the meridian.

Thus Bouguer, with a view of reconciling the degrees measured at the equator, in France and in Lapland, supposed the Earth to be a spheroid of revolution, in which the increase of the degrees, from the equator to the pole, was proportional to the fourth power of the sine of the latitude. It is found that this hypothesis does not satisfy the increase of the force of gravity from the equator to Pello.—An increase, which according to observation, is equal to forty-five ten millionths of the whole gravity, and which would be

only twenty-seven ten millionths in this hypothesis.

The above mentioned expressions give a direct and general solution of the problem which consists in determining the figure of a fluid mass in equilibrio, supposing it subjected to a motion of rotation, and composed of an infinity of fluids, of different densities, whose particles attract each other directly as their masses, and inversely as the squares of their distances.

Legendre had already solved this problem by a very ingenious analysis, which supposes the mass homogeneous. In this general supposition, the fluid necessarily takes the form of an ellipsoid of revolution, of which all the strata are elliptic, whose densities diminish at the same time that their ellipticities increase, from the centre to the surface.

The limits of compression of the whole ellipsoid, are $\frac{5}{4}$ and $\frac{1}{2}$ of the ratio of the centrifugal force, to the force of gravity at the equator. The first limit is relative

to the hypothesis of homogenity, and the second, to the supposition of the strata, infinitely near the centre, being infinitely dense, and consequently the whole mass of the spheroid acting as if concentrated in that point. In the latter case, the force of gravity being directed to a single point, and varying inversely as the square of the distance, the figure of the Earth would be such as has been above determined; but in the general hypothesis, the line which determines the direction of the force of gravity from the centre to the surface of the spheroid, is a curve, every element of which is perpendicular to the stratum through which it passes.

It is remarkable, that the variations observed in the length of the pendulum, follow pretty correctly the law of the squares of the cosines of the latitudes, at the same time that the variations in the measured degrees, differ very sensibly from this law. The general theory of the attractions of spheroids, affords a simple explanation of this phenomenon; it shews us that the

terms, which in the value of the terrestrial radius, differ from this law, become more sensible in the expression of the force of gravity, and still more sensible in the expression of degrees, where they may acquire a value sufficiently great to produce the phenomenon under consideration.

This theory likewise shews us, that the limits of the total increase of the force of gravity, taken at the equator as unity, are the products of 2 and $\frac{5}{4}$, by the ratio of the centrifugal force, to the force of gravity, the first limit referring to the case of an infinite density at the centre, the second to the case of homogenity. The increase, as derived from observation, being between these limits, indicates that the strata are more dense, as they approach the centre, which is conformable to the laws of hydrostatics. Thus the theory seems to accord with observation, as far as could be expected, considering our ignorance of the internal construction of the Earth.

The result of this agreement is, that in the calculation of the variations of the

force of gravity, and of parallax, we may consider the terrestrial meridians as of an elliptic form, the compression of which is the excess of the fraction $\frac{1}{115.2}$, above the total increase of the force of gravity from the equator to the poles.

The radius drawn from the centre of gravity of the terrestrial spheroid, to its surface at the parallel, the square of the sine of whose latitude is $\frac{1}{3}$, determines the sphere, whose mass is equal to that of the Earth, and whose density is equal to its mean density; this radius is 6369374 metres, and the force of gravity on this parallel, is the same as at the surface of this sphere.

But what is the proportion of the mean density of the Earth, to that of a known substance at its surface? The effect of the attractions of mountains, on the oscillations of pendulums, and on the direction of the plumb-line, may conduct us to the solution of this interesting problem.

It is true that the highest mountains are always very small, in proportion to the

Earth; but we may approach very near to the centre of their action, and this joined to the precision of modern observations, ought to render their effects perceptible.

The mountains of Peru, the highest in the world, seemed the most proper for this object. Bouguer did not neglect so important an observation in the journey which he undertook, for the measure of the meridional degrees at the equator.

But these great bodies being volcanic and hollow in their interior, the effect of their attraction was found to be much less than might be expected from their size. However it was perceptible; the diminution of the force of gravity at the summit of Pichincha, would have been 0.00149, without the attraction of the mountain, and it was observed to be 0.00118. The effect of the deviation of the plumb-line, from the action of another mountain, surpassed * 20″. Dr. Maskelyne has since

* 6″ 4.

measured, with great care, a similar effect, produced by the action of a mountain in Scotland: the result was, that the mean density of the Earth, is double that of the mountain, and four or five times greater than that of the common water. This curious observation deserves to be repeated several times on different mountains, whose interior construction is well known.

Let us apply the preceding theory to Jupiter.

The centrifugal force due to the motion of rotation of this planet, is nearly $\frac{1}{9}$ of the force of gravity at its equator; at least, if the distance of the fourth satellite from its centre, as given in the second Book, be adopted.

If Jupiter was homogeneous, the diameter of its equator might be obtained, by adding five-fourths of the preceding fraction to its shorter axis; taken as unity, these two axes would, therefore, be in the proportion of 41 to 36. According to observation, their proportion is that of 14 to 13. Jupiter, therefore, is not homogene-

ous. Supposing it to consist of strata, if the densities diminish from the centre to the surface, its ellipticity should be included between $\frac{5}{36}$ and $\frac{1}{18}$, the observed ellipticity being within these limits, proves the heterogenity of its strata, and by analogy that of the strata of the terrestrial spheroid, already rendered very probable in itself, and from the observations of the pendulum.

CHAP. VIII.

On the Figure of Saturn's Ring.

THE ring of Saturn, as has been shewn in the first Book, is formed of two concentric rings of very small thickness. By what mechanism do these rings sustain themselves round the planet? It is not probable that this should take place from the simple adhesion of their particles. Since, were this the case, the parts nearest to Saturn, sollicited by the constantly renewed action of gravity, would be at length detached from the rings, which would, by an insensible diminution, finally disappear, like all those works of nature which have not had sufficient force to resist the action of external causes. These rings support themselves then without effort, and only by the lines of equilibrium. But for this

it is requisite to suppose them possessed of a rotary motion round an axis perpendicular to their plane, and passing through the centre of Saturn, so that their gravitation towards the planet, may be balanced by the centrifugal force due to this motion.

Let us imagine a homogeneous fluid spread round Saturn in the form of a ring, and let us see what ought to be its figure, for it to remain in equilibrium, in consequence of the mutual attraction of its particles, of their gravitation towards Saturn, and their centrifugal force. If, through the centre of the planet, a plane is imagined to pass, perpendicular to that of the ring, the section of the ring by this plane, is what I shall call the *generating curve.* Analysis proves that if the magnitude of the ring is small in proportion to its distance from the centre of Saturn, the equilibrium of the fluid is possible when the generating curve is an ellipse of which the greater axis is directed towards the centre of the planet. The duration of the rota-

tion of the ring, is nearly the same as that of the revolution of a satellite, moved circularly at the distance of the centre of the generating ellipse. And this duration is about * four hours and a third, for the interior ring. Herschel has confirmed by observation this result, to which I had been conducted by the theory of gravitation.

The equilibrium of the fluid would also exist, supposing the generating ellipse variable in size and position, to the extent of the circumference of the ring; provided these variations are sensible only at a much greater distance than the axis of the generating section. Thus the ring may be supposed of an unequal breadth in its different parts, it may even be supposed of double curvature. These inequalities are indicated by the appearances and disappearances of Saturn's ring, in which the two arms of the ring have presented different phenomena. They are even neces-

* $10^h\ 32'$.

sary to maintain the ring in equilibrium round the planet, since if it was perfectly similar in all its parts, its equilibrium would be deranged by the slightest force, such as the attraction of a satellite, and the ring would finally precipitate itself upon the planet.

The rings by which Saturn is surrounded, are consequently irregular solids, of unequal breadth in the different points of its circumference, so that their centres of gravity do not coincide with the centres of their figure. These centres of gravity may be considered as so many satellites, moving round the centre of Saturn, at distances dependant on the inequalities of the rings, and with angular velocities equal to the velocities of rotation of their respective rings.

It may be imagined, that these rings, sollicited by their mutual action, by that of the Sun, and of the satellites of Saturn, ought to oscillate round the centre of this planet, and that their nodes, formed with the plane of the orbit of this

planet, should have a retrograde motion. It might be believed, that yielding to different forces, they should cease to be in the same plane; but Saturn having a rapid rotatory motion, and the plane of its equator being the same with that of its ring, and of its six first satellites, its action retains the system of these different bodies in the same plane. The action of the Sun, and of the seventh satellite, only changes the position of the plane of Saturn's equator, which in this motion carries with it the ring, and the orbits of the six first satellites, by a similar mechanism to that which retains the orbits of the satellites of Jupiter, and principally the orbit of the first, nearly in the plane of the equator of this planet.

Thus the constant position of Saturn's rings, and the orbits of the six first satellites in the same plane, indicate a considerable compression in this planet, and consequently a rapid motion of rotation, which has been confirmed by obser-

vation; and as all the satellites of Uranus move nearly in the same plane, we may conclude that this planet revolves upon itself, round an axis perpendicular to this plane.

CHAP. IX.

On the Atmosphere of the Celestial Bodies.

The thin, transparent, compressible, and elastic fluid which surrounds a body, and rests upon it, is called its atmosphere. We conceive a similar atmosphere surrounding every celestial body; the probability of its existence in all of them, is indicated by observation relative to the Sun and Jupiter. In proportion as the atmospherical fluid is elevated above the surface of a body, it becomes thinner, in consequence of its elasticity, which dilates it as it is less compressed. But if the particles of its surface were perfectly elastic, it would extend itself without ceasing, and finally would dissipate itself into space.

It is then requisite that the elasticity of

the atmospherical fluid should diminish in a greater proportion than the weight which compresses it; and that there may exist a state of rarity, in which it may be without elasticity. It should be in this state at the surface of the atmosphere.

All the atmospheric strata should take after a time the rotatory motion, common to the body which they surround. For the friction of these strata against each other, and against the surface of the body, should accelerate the slowest motions, and retard the most rapid, till a perfect equality is established among them. In these changes, and generally in all those which the atmosphere undergoes, the sum of the products of the particles of the body, and of its atmosphere, multiplied respectively by the area which their radii vectores projected on the plane of the equator, describe round their common centre of gravity, are always equal in equal time.

Supposing then, that by any cause whatever, the atmosphere should contract itself, or that a part should condense itself

on the surface of the body, the rotatory motion of the body, and of its atmosphere, would be accelerated, because the radii vectores of the area, described by the particles of the primitive atmosphere becoming smaller, the sum of the product of all the particles, by the corresponding area, could not remain the same, unless the velocity of rotation augments.

At its surface the atmosphere is only retained by its weight, and the form of this surface is such, that the force which results from the centrifugal and attractive forces of the body, is perpendicular to it. The atmosphere is flattened towards the poles, and distended at its equator, but this ellipticity has limits, and in the case where it is the greatest, the proportion of the axis of the pole and the equator, is as two to three.

The atmosphere can only extend itself at the equator, to that point where the centrifugal force exactly balances the force of gravity, for it is evident that beyond this limit, the fluid would dissipate itself. Relative to

the Sun, this point is distant from its centre by the length of the radius of the orbit of a planet, the period of whose revolution is equal to that of the Sun's rotation.

The Sun's atmosphere then does not extend so far as Mercury, and consequently does not produce the zodiacal light, which appears to extend even to the terrestrial orbit. Besides, this atmosphere, the axis of whose poles should be at least two-thirds of that of the equator, is very far from having the lenticular form which observation assigns to the zodiacal light.

The point where the centrifugal force balances gravity, is so much nearer to the body, in proportion as its rotatory motion is more rapid. Supposing that the atmosphere extends itself as far as this limit, and that afterwards it contracts and condenses itself from the effect of cold at the surface of the body, the rotatory motion would become more and more rapid, and the farthest limit of the atmosphere would approach continually to its centre: it will then abandon successively

in the plane of its equator, fluid zones, which will continue to circulate round the body, because their centrifugal force is equal to their gravity. But this equality not existing relative to those particles of the atmosphere, distant from the equator, they will continue to adhere to it. It is probable that the rings of Saturn are similar zones, abandoned by its atmosphere.

If other bodies circulate round that which has been considered, or if it circulates itself round another body, the limit of its atmosphere is that point where its centrifugal force, *plus* the attraction of the extraneous bodies, balances exactly its gravity. Thus the limit of the Moon's atmosphere, is the point where the centrifugal force due to its rotatory motion, plus the attractive force of the Earth, is in equilibrium with the attraction of this satellite. The mass of the Moon being $\frac{1}{597}$ of that of the Earth, this point is distant from the centre of the Moon, about the ninth part of the distance from

the Moon to the Earth. If, at this distance, the primitive atmosphere of the Moon had not been deprived of its elasticity, it would have been carried towards the Earth, which might have retained it. This is perhaps the cause why this atmosphere is so little perceptible.

CHAP. X.

Of the Tides.

If the investigation of the laws of the equilibrium of the fluids which cover the planets, presents great difficulties, that of the motion of these fluids agitated by the attractions of the heavenly bodies, offer still greater.

Thus Newton, who occupied himself the first with this important problem, was satisfied with determining the figure in which the ocean would remain *in equilibrio,* under the action of the Sun and Moon. He supposed that the sea, at every instant, took this figure; and this hypothesis, which extremely facilitates the calculations, gave him results, in many respects conformable with the observations. In fact, this great geometrician had re-

course to the action of the rotation of the Earth, to explain the retardation of the tides, beyond the passage of the Sun and Moon over the meridian; but his reasoning is unsatisfactory, and, moreover, appears contrary to the result of a rigorous analysis. The Academy of Sciences proposed this subject for a prize, in 1740; the successful pieces, contained the developement of the Newtonian theory, founded on the same hypothesis, of the ocean in equilibrium under the action of the attracting bodies. It is evident, nevertheless, that the rapidity of the Earth's motion prevents the waters that cover it, from taking at every instant, the figure suitable to the equilibrium of the forces, but the investigation of this motion, combined with that of the action of the Sun and Moon, was too difficult to be effected by the state of analysis at that time, and of the knowledge then possessed of the motions of fluids. But assisted by the discoveries which have since been made on both these subjects, I have again undertaken this

problem, the most intricate in celestial mechanics. The only hypotheses which I shall permit myself are, that the ocean inundates the whole Earth, and that it meets with but slight obstructions in its motion; the rest of my theory is rigorously exact, and founded on the principles of the motion of fluids. By thus conforming to nature, I have the satisfaction to see my results agree with the observations, particularly with respect to the small difference which subsists between the two tides of one day, which difference, according to the theory of Newton, should be very great. I obtained this remarkable result, namely, that to make this difference disappear, it is only necessary to suppose the ocean to have every where the same depth. Daniel Bernoulli, in his Essay on the Tides, which divided the prize of the academy, in 1740, endeavoured to explain this phenomenon, by supposing that the motion of the Earth was too rapid to permit the tides to accommodate themselves to the theory. But it can be shewn by analysis,

that this rapidity could not prevent the tides from being very unequal, if the depth of the ocean was not constant. We may see by this example, and by that of Newton, how much we should distrust the most plausible hypotheses, when not supported by rigorous calculation.

The preceding results, though very extensive, are still restricted by the supposition of a fluid, regularly spread over the Earth, and subject to very slight resistances in its motions. The irregularity in the depth of the ocean, the position and declivity of the shores, their situation relative to the neighbouring coasts, the friction of the waters against the bottom of the ocean, and the resistances they meet with: all these causes, which it is impossible to reduce to calculation, modify the oscillations of this great fluid mass. All that can be done is to analyse the general phenomena of the tides, which should result from the attractive forces of the Sun and Moon, and to derive from observation such data as are indispensably

necessary to complete the theory of the tides for each particular port. These data are so many arbitrary quantities depending on the extent of the sea, its depth, and the local circumstances of the port. Under this point of view, we shall consider the oscillations of the ocean, and their correspondence with observations.

Let us first consider the action of the Sun alone upon the ocean, and suppose its motion uniform in the plane of the equator. It is evident, that if the Sun acted on the centre of gravity of the Earth, and of every particle of the ocean, by exerting equal and parallel forces, the whole system of the terrestrial spheroid would obey these forces by a common motion, and the equilibrium of the waters would not be at all altered. This equilibrium, then, is only deranged by the difference of these forces, and by the inequality of their directions. A particle of the ocean, placed directly under the Sun, is more attracted than the centre of the Earth. It tends,

therefore, to separate itself from it, but it is retained by its gravity, which this tendency diminishes. Twelve hours afterwards, this particle is opposite to the Sun, which attracts it less forcibly than it does the centre of the Earth; the surface of the terrestrial globe therefore tends to separate itself from it, but the gravity of the particles retains it. This force is therefore diminished also in this case by the solar attraction. But since the distance of the Sun is very great, compared with the radius of the Earth, it is easy to see that the diminution of gravity in each case is very nearly the same. A simple decomposition of the action of the Sun upon the particles of the ocean, is sufficient to shew, that in any position of this body, relatively to these particles, its action in disturbing their equilibrium, becomes the same after twelve hours. And it may be established as a general principle in mechanics, that the state of a system of bodies, in which the primitive conditions of

motion have disappeared by the resistances it meets with, is periodic, like the forces which solicit it. The state of the ocean should therefore be the same at each interval of half a-day, so that the tide should ebb and flow in this interval.

The law according to which the water rises and falls, may be thus determined. Let us conceive a vertical circle, whose circumference represents half a day, and whose diameter is equal to the whole tide, or the difference between the height of high and low water, and let the arcs of this circumference, reckoning from the lowest point, express the time elapsed since low water, the versed sines of these arcs will express the heights of the water, corresponding to these times. Thus, the ocean in rising, covers in equal times, equal arcs of this circumference. This law is exactly observed in the middle of the ocean, which is free on every side, but in our harbours, local circumstances produce some deviations. The sea employs rather a longer time to fall than to rise,

which difference at Brest amounts to about * $10\frac{1}{2}$ minutes.

The greater the extent of the surface of the water, the more perceptible are the phenomena of the tides. In a fluid mass, the impressions which a fluid particle receives, are communicated to the whole. It is thus that the action of the Sun, which is insensible on an insulated particle, produces on the ocean such remarkable effects. Let us imagine, at the bottom of the sea, a curved canal, terminated at one of its extremities by a vertical tube, rising above the surface of the water, and which, if prolonged, would pass through the centre of the Sun.

The water will rise in this tube by the direct action of the Sun, which diminishes the gravity of its particles, and particularly by the pressure of the particles enclosed in the canal, which all make an effort to unite themselves beneath the Sun. The elevation of the water in the tube

* 14′ 27″.

above the natural level of the sea, is the integral of all these infinitely small efforts. If the length of this canal is increased, this integral also becomes greater, because it extends over a larger space, and because there will be a greater difference in the quantity and direction of the forces, by which the extreme particles are sollicited.

By this example we see the influence which the extent of the sea has upon the phenomena of the tides, and the reason why they are insensible in the very small seas, as the Euxine and the Caspian. The magnitude of the tides depends also much on local circumstances. The oscillations of the ocean, when confined in a narrow channel, may become extremely great, and these may be augmented by the reflection of the waters from the opposite shore. It is thus, that the tides, very small in the South Sea islands, are very considerable in our harbours.

If the ocean covered a spheroid of revolution, and experienced no resistance to its motion, the instant of high water would

be that of the passage of the Sun over the superior or inferior meridian; but it is not thus in nature; local circumstances produce great variations in the times of high water, even in harbours that are very near each other. To have a just idea of these variations, we may suppose a large canal communicating with the sea, and extending into the land; it is evident that the undulations which take place at its entrance, will be propagated successively through its whole length, so that the figure of its surface will be formed by the undulations of large waves in motion, which will be incessantly renewed, and will describe the whole length of the interval of half-a day. These waves will produce at every point of the canal, a flux and reflux, which will follow the preceding laws, but the hours of the flowing will be retarded, in proportion as the points are further from the entrance of the canal. What we have here said of a canal, may be applied to rivers whose surfaces rise and fall by similar waves, notwithstanding the contrary motion of their waves. These waves are observed

in all rivers near their *embouchure*. They are propagated to great distances in great rivers; and at the strait of Pauxis, in the river of the Amazons, it is sensible at two hundred leagues from the sea. Let us next consider the action of the Moon, which we will suppose to move uniformly in the plane of the equator. It is evident that it must excite in the ocean, an ebb and flow similar to that resulting from the action of the Sun, and whose period is half a lunar day. Now it has been shewn in the preceding Book, *that the total motion of a system agitated by very small forces, is equal to the sum of the partial motions, which every force would have impressed separately*; the two partial tides, therefore, produced by the action of the Sun and Moon, combine without deranging each other, and from their combination results the tides, which we observe in our ports.

From hence arise the most remarkable phenomena of the tides. The instant of the lunar tide is not the same with that of

the solar tide, since their periods are different. If these two tides coincide, the following lunar tide will retard upon the solar tide, by the excess of half a lunar day, above half a solar day, that is * 1752″ 5. These retardations accumulating from day to day, the full lunar tide will finish by coinciding with the low solar tide, and reciprocally. When the lunar and solar tides coincide, the combined tide is the greatest, which happens about the syzigies. The combined tide is the least, when the full tide, relative to one of the bodies, coincides with the low tide of the other, which is the reason of the tides being least at the quadratures. If the solar tide exceeded the lunar tide, it is clear that the hours of the greatest and least combined tides, would coincide with the times at which the solar tide would happen, if it alone existed. But if the lunar *tide exceeds the solar tide, then the least* combined tide coincides with the low solar

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* 9′ 27″ 8.

tide, and consequently its time is a quarter of a day's interval from the hour of the greatest combined tide. This offers an easy method of deciding, if the lunar tide is greater or less than the solar tide. All the observations concur in making the hour of the least tides differ by a quarter of a day from that of the greatest tide, which prove that the lunar tide exceeds the solar tide.

We have seen, in the first Book, that the mean value of the greatest tide in every month, is nearly $5^{me}.888$, and that the mean least tide is $2^{me}.789$. It is easy to conclude, after the requisite reductions, that the mean lunar tide, that which corresponds to the constant part of the parallax of the Moon, is three times less than the mean solar tide; or, in other words, that the action of the Moon, to elevate the waters of the ocean, is three times as great as that of the Sun.

The extent of the variation of the total tides, taken at the maximum, or minimum, is exactly the same by the theory

of gravity, as by observation. Their increase, in departing from the minimum, is the double of their diminution, in departing from the maximum, as the observations indicate. Since the lunar tide exceeds the solar tide, the combined tide should be regulated chiefly by the lunar tide, and in a given time there should be as many *tides as passages of the Moon* over the superior or inferior meridian, which is conformable with what we observe. But the instant of the combined tide should oscillate round the instant of the lunar tide, according to some law depending on the phases of the Moon, and of the ratio of its action, to that of the Sun. The first of these instants precedes the second, from the greatest to the least tides, and follows it from the least to the greatest, so *that* the mean *time* of the combined tide being the same as the lunar tide, the mean retardation of the tides in one day is * 3505″.

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* 18′ 55″ 6.

According both to theory and observation, the height of the tides, and their retardation, vary according to the phases of the Moon. The least retardation coincides with the greatest height, and the greatest retardation with the least height, and the theory, by a remarkable coincidence, gives these retardations * 2705″ and † 5207″, the same as results from observation.

This agreement proves the justness of the theory, and the exactness of the supposed ratio between the actions of the Sun and the Moon. In changing this ratio a small quantity, a great discordance would arise in the heights and retardations, which therefore are capable of giving us this ratio with great precision.

We may here make an important remark, on which depends the explanation of several phenomena relating to the tides. If the spheroid which the sea covers, was a solid of revolution, the partial tides

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* 14′ 36″ 4.   † 28′ 7″.

would take place at the instant of the passages of their respective bodies over the meridian. Thus, when the sysygies happened at noon, the two tides, lunar and solar, would coincide with this instant, which would be that of the greatest combined tide. This greatest tide would take place at the same day as the sysigy; if the two partial tides followed nearly by the same interval, the passage of their respective bodies over the meridian. But the daily motion of the Moon in its orbit being considerable, the rapidity of this motion may influence very sensibly the interval between the passage of the Moon and the lunar tide.

We may form a just idea of this phenomenon, by imagining as above, a vast canal communicating with the ocean, and advancing very far into the continent, under the meridian of its *embouchure*. If we suppose that at this *embouchure*, the full tide takes place at the instant of the passage of the heavenly body over the

meridian, and that it employs * twenty-one hours to arrive at its extremity, it is evident that at this last point, the solar tide will happen one hour after the passage of the Sun over the meridian. But two lunar days forming † 2.070 solar days, the lunar tide will only be ‡ 30′ later than the passage of the Moon; thus there will be § 70′ difference in the intervals of the solar and lunar tides, after their respective passages over the meridian.

From hence it follows that the maximum and minimum of the tides does not take place on the very days of the sysigy and quadrature, but one or two days after, when the interval of the lunar tide after the passage of the Moon, added to the interval of the passage of the Moon after that of the Sun, is equal to the interval of the solar tide, after the passage of the Sun. Thus, in the preceding example, the maximum and minimum, which, at the *embouchure* of the canal,

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* $50^h\ 24'$. † $2^d\ 40'\ 48.''$ ‡ $16'\ 12.''$ § $37'\ 48''$.

take place the day of the sysygy and quadrature, will not arrive at its extremity till * twenty-one hours afterwards.

I have found, by comparing a great number of observations, by different methods, that at Brest the interval by which the greater tide follows the sysygy, is very nearly a day and a half. Hence it follows, that in this port the solar tide follows the passage of the Sun, † 18358″ and that the lunar tide follows the passage of the Moon ‡ 13101″. The hours of the tide at Brest are therefore the same as at the extremity of a canal, which should communicate with the ocean, if we suppose that at its *embouchure*, the partial tides take place at the instant of the passage of their respective bodies over the meridian, and that they employ a day and a half to arrive at its extremity, supposed 18358″ more to the eastward, than its *embouchure*. And in general, both observation and theory have shewn me, that each of the ports in

* $2^d\ 2^h\ 24'$. † $4^h\ 24'\ 21''$. ‡ $3^h\ 8'\ 39''$.

France, may be considered relatively to the tides, as the extremity of a canal, at whose *embouchure* the partial tides take place at the very instant of the passage of the Sun and Moon, and are transmitted in a day and a half to its extremity, supposed situated to the eastward of its *embouchure*, by a quantity very different for the different ports.

It may be observed that the difference in the intervals, by which the partial tides follow the meridian passages of the bodies which produce them, do not essentially change any of the phenomena of the tides. For a system of bodies moving in the equator, it only retards one day and a half the phenomena, which would take place by calculation from an hypothesis, in which these intervals are nothing. Many philosophers have attributed the retardation of the phenomena of the tides, relatively to the phases of the Moon, to the time employed by its action, to transmit itself to the Earth. But this hypothesis is incompatible with the inconceivable

activity of the attractive force, the proof of which will be shewn at the end of the Book; it is not therefore to the time employed in this transmission, but to that necessary to communicate the original impressions through the ocean to our ports, that we are to attribute this delay.

The power of a celestial body to raise a particle of water placed between it and the centre of the earth, is equal to the difference of its action on the centre and on the particle; and this difference is double the quotient of the mass of the heavenly body, multiplied by the terrestrial radius, and divided by the cube of the distance of the centres of the celestial body and the Earth. This quotient relatively to the Sun, is by the Fifth Chapter, the hundred and seventy-ninth part of the force of gravity, which solicits the Moon towards the Earth, multiplied by the proportion of the terrestrial radius, to the distance of the Moon; this force of gravity is very nearly equal to the sum of the masses of the Earth and Moon, divided by the square of the

lunar distance; the power of the Sun to raise the waters of the sea is therefore eighty-nine times and a half less than the sum of the masses of the Earth and Moon, multiplied by the terrestrial radius and divided by the cube of the lunar distance.

But this force according to observation, is only a third of the force of the Moon, which is equal to double its mass, multiplied by the terrestrial radius, and divided by the cube of its distance; thus the mass of the Moon is to the sum of the masses of the Earth and Moon as 3 is to 179; from whence it follows that this mass is very nearly $\frac{1}{58.7}$ of that of the Earth. Its volume being only $\frac{1}{49.316}$ of that of the Earth, its density is 0.8401, the mean density of the Earth being taken as unity; and the weight which on the Earth is unity, transported to the surface of the Moon, would be reduced to 0.2291.

Nevertheless the irregularity in the depth of the sea, which, as has been shewn, produces a perceptible difference in the interval by which the lunar and solar

tides follow the transits of their respective bodies over the meridian, may likewise influence the proportional altitudes of these two tides.

Let us imagine a port situated at the junction of two canals, communicating with the sea under the same meridian; let us also suppose that at their *embouchure,* the partial tide of each celestial body, happens at the very instant of its transit over the meridian. The tide in the port will be the result of the tides transmitted to it by each canal; if the tide employs one day to pass from the sea to the port by the first canal, and eight days and a half by the second, the difference of these intervals, being seven days and a half, the two solar tides of each canal will coincide in the port, and the compound solar tide will be equal to their sum. But as seven solar days and a half only produce seven lunar days and a quarter; the full lunar tide of the first canal should coincide with the last lunar tide of the second; thus the lunar tide in the port will be

only the difference of the lunar tides, transmitted by the two canals. Supposing therefore that at the *embouchures* the tides may be proportional to the force of the celestial bodies, they will cease to be so in the port, where it may even happen that the lunar tide may be weaker than the solar.

It is important, therefore, when we wish to ascertain the proportional forces of the Sun and Moon from the phenomena of the tides, to be assured that the observed tides are in proportion to these forces. Analysis furnishes different means for this object; applying them to the observations made at Brest, I found that this proportion existed in a very approximate manner; thus the value which we have assigned to the mass of the Moon, should differ very little from the true value.

Hitherto we have supposed the Sun and Moon moved uniformly on the plane of the equator. Let us now vary their motions and their distances from the centre of the Earth. In developing the

expressions of their actions upon the ocean, we may represent each term by the action of a body moved uniformly in a circle round the Earth, it will then be easy, by the principles already explained, to determine the flux and reflux of the ocean, corresponding to the different inequalities of the Sun and Moon. In submitting in this manner the phenomena of the tides to analysis, it is found that the tides produced by the Sun and Moon augment in an inverse ratio to the cubes of their distances. The tides ought therefore, *ceteris paribus*, to increase in the perigee and diminish in the apogee of the Moon. This phenomenon is extremely apparent at Brest; by examining the observations, I find that * 100″ of variation in the semi-diameter of the Moon, answers to half a metre of variation in the total tide, when the Moon is in the equator; and this result of observation is so conformable to that given by the theory, that we might from this alone have found

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* 32″4.

the law of the variation of the Moon's action relative to its distance. The variations in the distance of the Sun from the Earth, are sensible in the heights of the tides, but in a much less degree than those of the Moon, because its action *in elevating the waters* is three times less, and its distance from the Earth varies in a less ratio. This result of the theory is also verified by observation. The action of the Moon being greater and its motion more rapid, when it is nearer the Earth, the combined tide in the sysygies perigee, ought to approximate to the lunar tide, which ought itself to approximate to the passage of the Moon; for we have seen that the partial tide approaches so much nearer to the body that causes it as its motion is more rapid, the tide's perigee on the day of the sysygy ought therefore to advance, and the tide's apogee to retard. It has been mentioned in the First Book that according to observation, every † minute of

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* 32″4.

increase or diminution in the lunar semi-diameter advances or retards the high tide by * 354″, which is very nearly what results from the theory. The parallax of the Moon influences also the interval of two consecutive tides of the morning and evening, about the sysygies, or in the vicinity of the maximum of the tides. According to the theory a minute of variation in the semi-diameter of the Moon, produces a variation in this interval of †258″, which is exactly confirmed by observation.

This phenomenon equally takes place at the quadratures, but the theory shews that it is three time less than at the sysygies, and this answers to the observations. To conceive this we must recollect that the daily retardation of the lunar tide, augments as the Moon's motion is the more rapid, as happens at the perigee; and that the retardation of the tides at the sysygies augments and approaches the

* 1′54″. † 1′23″.

daily retardation of the lunar tide, when the lunar force augments; these two causes, therefore combine in augmenting the interval of the tides at the sysigies' perigee. At the quadratures, when the lunar force increases, the daily retardation of the tide diminishes, by approaching the retardation of the lunar tide; thus the interval of the tides is increased by the rapidity of the motion of the Moon perigee, and diminished by the increase of the lunar force: these two causes, therefore, acting, then, in opposite directions, the increase in the retardation of the tide is only the effect of their difference, and for this reason is less than in the sysygies. Thus, having developed the theory of the tides, upon the supposition that the Sun and Moon move in the plane of the equator, we shall next consider the motion of these bodies, such as they really are in nature, and we shall see that new phenomena arise from their change of declination, which, compared with observation, tend still more to confirm the preceding theory.

This complicated case may be reduced to that of several bodies moved uniformly in the plane of the equator, but we must give to these bodies very different motions in their orbits. Some moving very slowly, produce a flux and reflux, whose period is half a day, others have a revolution equal to half the revolution of the earth, and they produce a flux and reflux, whose period is a day. Others have a revolution nearly equal to the *rotation of the Earth*. They produce a flux and reflux whose periods are of a month and of a year.

Let us examine these three species of tides.

The first contains not only the oscillations which we have considered above, and which depend on the motion of the Sun and Moon, and on the variations of their distances from the Earth, but likewise others depending on their declinations. In submitting these to analysis, we find, that the total tides of the equinoctial sysygies are greater than those of the sol-

stitial sysygies, in the ratio of radius, to the square of the cosine of the declination of the Sun, or of the Moon, about the period of the solstices; we find, moreover, that the tides of the solstitial quadratures exceed those of the equinoctial quadratures, in a greater ratio than that of radius to the square of the cosine of the declination of the Moon, about the equinoctial quadratures. All the observations confirm these theoretical results, and leave no doubt of the diminution of the action of the bodies, as they deviate more or less from the equator.

The declinations of the Sun and Moon sensibly affect even the law of the diminution and augmentation of the tides, reckoning from the *maximum* or *minimum*. Both by theory and observation, their diminution is about one-third more rapid in the equinoctial sysygies, than in the solstitial. Their increase, both by theory and observation, is about twice as rapid in the equinoctial quadrature, as in those of the solstice.

The position of the nodes is likewise sensible in the height of the tides, by its influence on the declinations of the Moon.

The motion of the Moon, in right ascension, being more rapid in the solstices than at the equinoxes, must bring the lunar tide nearer to the Moon's meridian passage; the hour of the tides in the equinoctial sysygies should retard upon the hour of the solstitial sysygies; for the same reason, the hour of the tide, at the solstitial quadratures, should retard on that of the equinoctial quadratures; the theory gives this latter retardation about quadruple that of the first.

The declinations of the Sun and Moon influence likewise the daily retardation of the equinoctial and solstitial tides; it should be greater about the solstitial sysygies, than at the equinoctial, and still greater about the equinoctial quadrature, than at the solstitial; and in this second case, the difference in the retardation is four times greater than in the first: and observation confirms, with remarkable pre-

cision, all these theoretical results. The tides of the second class, whose period is a day, are proportional to the product of the sine, by the cosine of the declination of the body. They are nothing when it is in the equator, and they increase as it departs from it. By being combined with the tides of the first class, they render the two tides of the same day unequal. It is for this reason, that the morning tide at Brest is about $0^{me}.183$ greater than the evening tide, about the sysygies of the winter solstice, and less by the same quantity about the summer solstice, as has been observed in the First Book: for the same cause the morning tide is greater by $0^{me}.136$, about the equinoctial quadratures of autumn, and less by the same quantity at the equinoctial quadratures in the spring.

In general, the tides of the second class are not very considerable in our harbours, their magnitude is an arbitrary quantity, depending on local circumstances, which may augment them, and diminish at the same time the tides of the first class, so

as to render the former insensible. For, let us conceive a large canal communicating by its two extremities with the ocean, the tide in a port situated on the border of this canal, will be the result of the undulations transmitted through its two *embouchures.* Now it may happen that the undulations of the first class arrive at such a time, that the maximum of one may coincide with the minimum of the other, and if these should be equal, it is clear that there would be neither flux nor reflux in this port, in consequence of these undulations. But there will be a flux produced by the undulations of the second class, which, having a period twice as long, will not correspond in such a manner, as for the maximum of those which enter by one embouchure, to coincide with the minimum of those which enter by the other.

In this case, there will be no tide when the Sun and Moon are in the plane of the equator; but it will become sensible when the Moon deviates from this plane, and then there will be one flux and one reflux

in each lunar day, so that if the flux arrives at the setting of the Moon, the reflux will take place when it rises. This singular phenomenon has been observed at Batsha, a port in the kingdom of Tunquin, and in some other places. It is probable, that observations made in the different parts of the world, would afford all the intermediate varieties between the tides of Batsha, and those of our ports.

Let us consider, finally, the tides of the third class, whose periods are very long, and independant of the rotation of the Earth. If the length of this period was infinite, these tides would have no further effect than to change the permanent figure of the ocean, which would soon arrive at that state of equilibrium due to the forces which produce them. But it is evident that the length of these periods will produce nearly the same effect on the tides, as in the case of its being infinite. We may therefore consider the ocean as constantly *in equilibrio*, under the action of fictitious bodies, which produce tides of

the third class, which may be determin in this hypothesis. These tides are ve small, but are nevertheless sensible Brest, and correspond to the result of ca culations.

I have entered into a long detail on th tides, because it is the nearest and most pe ceptible result of the celestial attractio to us, and one most worthy the attentic of philosophers. We see, by the expos tion which I have made, the agreement c the theory of the tides, founded on th law of universal gravitation, with th phenomena of the heights and interval c the tides. If the Earth had no satellite if its orbit was circular, and situated o the plane of the equator, we should onl have had, to have enable us to recogniz the action of the Sun upon the ocean, th hour of high water always the same, an the law of its formation. But the action c the Moon, combining with that of the Sur produces in the tides varieties relative t its phases, which, by their agreement wit observation, give a great probability t

the truth of the theory of gravitation. All the inequalities of motion, produced by the declinations and distances of these two bodies, give rise to a number of phenomena, which, being recognized by observation, place this theory out of the shadow of doubt. It is thus that the varieties in the action of causes, establish their existence.

The action of the Sun and Moon on the Earth, a necessary consequence of the universal attraction, demonstrated by all the celestial phenomena, being directly confirmed by the phenomena of the tides, ought to leave no uncertainty on the subject. It is indeed brought now to such a degree of perfection, that not the least difference of opinion exists upon the subject, among men sufficiently learned in the science of geometry and mechanics, to comprehend its relation with the law of universal gravitation.

A long series of observations, more precise than have hitherto been made, will rectify the elements already known, and

fix the value of those which are uncertain; and develope phenomena which before were obscured in the errors of observation. The tides are not less interesting to understand, than the inequalities of the heavenly bodies, and equally merit the attention of observers. We have hitherto neglected to follow them with sufficient precision, because of the irregularities they present. But I can assert, after a careful investigation, that these irregularities disappear by multiplying the observations; nor is it necessary that their number should be extremely great, particularly at Brest, whose situation is very favourable for this species of observation.

I have now only to speak of the method of determining the time of high water, on any day whatever. We should recollect, that each of our ports may be considered as the extremity of a canal, at whose *embouchure* the partial tides happen at the moment of the passage of the Sun and Moon over the meridian, and employ a day and a half to arrive at its extremity,

supposed eastward of its embouchure, by a certain number of hours.—This number is what I call the fundamental hour of the port. It may easily be computed from the hour of the establishment of the port, by considering this as the hour of the full tide, when it coincides with the sysygy. The retardation of the tides, from one day to another, being then * 2705″, it will be † 3951″ for one day and a half, which quantity is to be added to the hour of the establishment, to have the fundamental hour. Now, if we augment the hours of the tides at the *embouchure* by ‡ 15 hours, *plus* the fundamental hours, we shall have the hours of the corresponding tides in our ports. Thus, the problem consists in finding the hours of the tides in a place whose longitude is known, on the supposition that the partial tides happen at the instant of the passage of the Sun and Moon over the meridian. For

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* 14′ 36″. † 21′ 20″. ‡ 36ʰ.

this purpose analysis affords very simple formulæ, which are easily reduced to tables, and very useful to be inserted in the ephemerides that are destined for navigators.

CHAP. XI.

On the Stability of the Equilibrium of the Ocean.

SEVERAL irregular causes, such as hurricanes and earthquakes, agitate the sea, elevate it to a great height, and sometimes oblige it to forsake its limits. Nevertheless, observation shews us that it has a *tendency to return to its former state* of equilibrium, and that the friction and resistances of all kinds that it experiences, would very soon bring it to this state, without the action of the Sun and Moon. This tendency constitutes the stable equilibrium, which we mentioned in the Third Book. We have there shewn that the stability of the equilibrium of a system of bodies may be absolute, or take place, whatever small derangement it may receive; or it may be relative, and depend

on the nature of the primitive disturbance. To which class belongs the stability of the ocean? This is what observation cannot teach us with absolute certainty; for, although in the almost infinite variety of disturbances to which the ocean is liable, from the action of irregular causes, it may appear to return to its former state of equilibrium; yet we may nevertheless apprehend, that some extraordinary cause may communicate to it a shock, which though inconsiderable at its origin, may augment continually, and elevate it above the highest mountains: this would explain several phenomena in natural history. It is therefore interesting to investigate the conditions which are necessary for the absolute stability of the ocean, and to examine if these conditions exist in nature. In submitting this object to analysis, I have assured myself that this equilibrium is stable, if its density is less than the mean density of the Earth, which is extremely probable, for it is natural to think, that the strata are more dense as

they approach the centre. We have besides seen, that this is proved by experiments on pendulums, by the measurement of degrees, and by the attractions of mountains. It appears then, that the equilibrium of the ocean is stable, and if, (as seems certain) the waters have formerly covered continents, which at present are elevated much above its level, we must not search for the cause in the want of stability in their equilibrium. I have likewise discovered, by the means of analysis, that this stability would cease to exist, if the mean density of the sea exceeded that of the earth, so that the stability of the equilibrium of the ocean, and the excess of the density of the terrestrial globe above that of the waters which cover it, are reciprocally connected one with the other.

CHAP. XII.

Of the Oscillations of the Atmosphere.

To arrive at the ocean, the action of the Sun and Moon must traverse the atmosphere, which must necessarily be subject to their influence, and experience similar oscillations to those of the ocean. From hence arise winds and variations in the barometer, the periods of which are the same as those of the flux and reflux of the ocean. But these winds are very inconsiderable, and almost insensible, in an atmosphere so much agitated by other causes. The extent in the oscillations of the barometer, is only one millimetre at the equator, where it is the greatest. Nevertheless, as local circumstances may considerably augment the oscillations of the ocean, they may equally increase the os-

cillations of the barometer, the observation of which merits the notice of philosophers.

We may here remark, that the action of the Sun and Moon, produce neither in the ocean nor in the atmosphere, any motion from east to west. That which is observed in the atmosphere, between the tropics, under the appellation of the trade-winds, proceeds therefore from some other cause —this seems to be the most probable.

The Sun, which we will suppose, for the sake of simplicity, in the plane of the equator, there rarifies by its heat the columns of air, and elevates them above their natural level, they should then re-descend by their weight, and be carried towards the poles in the superior part of the atmosphere, but at the same time, a current of cool air should arrive from the climates near the poles, to replace that which has been rarefied at the equator. Thus, two opposite currents of air are established, one in the inferior, the other in the superior part of the atmosphere. But

do under the covenant of works?

Feb. 1. Walked with M. to the British Museum, and though I saw there much for which I could at times glorify God,—as the varieties of birds, fishes, reptiles, minerals, &c. and the works of his intelligent creatures,—I was plagued with the workings of an evil, selfish, dissipated, discontented heart.

3. Went to bed with an earnest and hopeful desire of living in poverty of spirit and a sense of my own unworthiness.

4. The temper I wished to retain was a source of great tranquillity to me this morning. I was rather oppressed with care, yet I checked the suggestions of sloth by considering the example of Christ and his ministers in the present day, and was rather humbled

If we consider all the causes which disturb the equilibrium of the atmosphere, its great mobility arising from its fluidity and elasticity; the influence of heat

rection of the meridians; we should not be surprized at the inconstancy and variety of its motions, which it would be very difficult to subject to any fixed and certain laws.

the axis of the poles, and of a meniscus surrounding the sphere, and whose greatest thickness corresponds with the equator of the spheroid. The particles of this meniscus may be considered as so many small moons adhering together, and which make their revolutions in a period equal to the revolution of the Earth on its axis.

The nodes of all their orbits should therefore have a retrograde motion, arising from the action of the Sun, in the same manner as the nodes of the lunar orbit; and from the connection of these bodies together, there should succeed a retrograde motion of the whole meniscus; but this meniscus divides its retrograde motion, with the sphere to which it is attached, which, for this reason, becomes slower; the intersection of the equator and the ecliptic, that is to say, the equinoctial points, should have a retrograde motion. *Let us endeavour to investigate both the law and the cause of this phenomena.

And first we will consider the action of the Sun upon a ring, situated in the plane

of Revelations, and so very lively was the impression on my mind, that I was often in tears. So awful, so awakening is this book to me! Prayed with more fervour than I have done of late, and went to bed full of the sense of the importance of eternal things.

6. In the morning I sought to rouse myself to greater earnestness in prayer. It was my earnest desire to walk in the fear of God's holy name, and to have a more awful alarm about my state, and to dread his displeasure. Looked at an iron foundry in Wall's Lane: the fierce fire raised many solemn ideas of God's power, and of hell.

7. At church this morning I began to read the

all the time continuing stationary; and this would be the case but for the motion of the ring, which we now suppose to turn round in the same time as the Earth. By this motion, the ring is enabled to preserve a constant inclination to the ecliptic, and to change the effect of the action of the Sun into a retrograde motion of the nodes. It gives to the nodes a variation, which otherwise would be in the inclination, and it gives to the inclination a permanency, which otherwise would rest with the nodes. To conceive the reason of this singular effect, let us suppose the situation of the ring varied an infinitely small quantity, in such a manner that the planes of its two positions intersect each other, in a line perpendicular to the line of nodes.

At the end of any instant whatever, we may decompose the motion of each of its points into two, one of which should subsist alone in the following instant, the other perpendicular to the plane of the ring, and which should be destroyed. It

is evident that the resulting force of these second motions, relative to all the points of the upper part of the ring, will be perpendicular to its plane, and placed on the diameter which we just now considered, and this is equally true for the lower part of the ring. That this resulting force may be destroyed by the action of the solar orbit, and that the ring, by virtue of these forces, may remain in equilibrio on its centre, it is requisite that these forces should be contrary to each other, and their moments, relatively to this point, equal. The first of these conditions requires that the change of position, supposed to be given to the ring, be retrograde; the second condition determines the quantity of this change, and consequently the velocity of the retrograde motion of the nodes. And it is easily demonstrated, that this velocity is proportional to the mass of the Sun, divided by the cube of the distance from the Earth, and multiplied by the cosine of the obliquity of the ecliptic.

Since the planes of the ring, in its two

consecutive positions, intersect each other in a diameter perpendicular to the line of its nodes, it follows that the inclination of these two planes to the ecliptic, is constant, and the inclination of the ring does not vary, by the mean action of the Sun.

That which has been explained relatively to a ring, may be demonstrated by analysis, to hold true in the case of a spheroid, differing but little from a sphere. The mean action of the Sun produces in the equinoxes a motion proportional to its mass, divided by the cube of its distance, and multiplied by the cosine of the inclination to the ecliptic. This motion is retrograde when the spheroid is flattened at the poles; its velocity depends on the compression of the spheroid, but the inclination of the equator to the ecliptic, always remains the same.

The action of the Moon produces likewise a similar retrogradation of the nodes of the terrestrial equator, in the plane of its orbit; but the position of this plane and its inclination to the equator inces-

santly varying, by the action of the Sun and the retrograde motion of the nodes produced by the action of the Moon, being proportional to the cosine of this inclination, this motion is variable.

Besides, in supposing it uniform, it would, according to the position of the lunar orbit, cause a variation both in the retrograde motion of the equinoxes, and in the inclination of the equator to the ecliptic. A calculation, by no means difficult, is sufficient to show, that the action of the Moon, combined with the motion of the plane of its orbit, produces 1. A mean motion in the equinoxes, equal to that which it would produce if it moved in the plane of the ecliptic. 2. An inequality subtractive, from this retrograde motion, and proportional to the sine of the longitude of the ascending node of the lunar orbit. 3. A diminution in the obliquity of the ecliptic, proportional to the cosine of this same angle. These two inequalities are represented at once by the motion of the extremity of the terrestrial axis (pro-

longed to the heavens) round a small ellipse, conformably to the laws explained in Chap. XI. of Book I.

The greater axis of this ellipse is to the lesser, as the cosine of the obliquity of the ecliptic is to the cosine of double this obliquity. We may comprehend from what has been said, the cause of the precession of the equinoxes, and of the nutation of the Earth's axis, but a rigorous calculation, and a comparison of its results with observation, is the true test of the truth of a theory. That of universal gravitation is indebted to d'Alembert, for the advantage of having been thus verified in the case of the two preceding phenomena. This great mathematician first determined by a beautiful analysis the motions of the axis of the Earth, by supposing the strata of the terrestrial spheroid to be of any density or figure whatever, and he not only found his results exactly conformable to observation, but obtained an accurate determination of the dimensions of the small ellipse described by the pole of the

Earth, as to which the observations of Bradley had left some little doubt.

The influence of a heavenly body, either upon the motion of the terrestrial axis of the Earth, or upon the ocean, is always proportional to the mass of that body, divided by the cube of the distance of that body from the Earth. The nutation of the Earth's axis being due to the action of the Moon alone, while the precession of the equinoxes arises from the combined action of the Sun and Moon, it follows that the observed values of these two phenomena, should give the ratio of their respective actions. If we suppose, with Bradley, the annual precession of the equinoxes to be *154″4, and the whole extent of the nutation †55″6, the action of the Moon is found to be double that of the Sun. But a very small difference in *the extent of the nutation*, produces a very considerable one in the ratio of the actions of these two bodies, to make it equal three

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* 50″. † 17″9.

to one, as indicated by the observations of the tides, it is sufficient to suppose the extent of the nutation *62.2. Dr. Maskelyne, by re-examination of the observations of Bradley, finds this quantity †58″6, which differs but ‡3″6 from the result obtained by the phenomena of the flux and reflux of the ocean. So small a quantity being nearly insensible in the observations of the fixed stars, the ratio of the solar and lunar action is better determined by that of the tides; it seems to me, therefore, that we should fix the equation of the nutation at §31″1, that of the precession at 58″2, and the lunar equation of the tables of the Sun, ¶27″5. The phenomena of the precession and of the nutation throws a new light on the constitution of the terrestrial spheroid. They gave a limit to the compression of the earth supposed elliptic, hence it appears that this compression does not exceed $\frac{1}{305}$, which accords with the experiments that have

* 20″ 1. † 18″ 9. ‡ 1″ 1. § 10″. ¶ 8″ 9.

been made on pendulums. We have seen in Chap. VII. that there exists in the expression of the radius vector of the terrestrial spheriod, terms, which, but little sensible in themselves, and on the length of the pendulum, cause the degrees of the meridian to deviate considerably from the elliptic figure. These terms disappear entirely in the values of the precession and nutation, and for this reason, these phenomena agree with the experiments on pendulums. The existence of these terms, therefore reconciles the observations of the lunar parallax, those of the pendulums and degrees of the meridian, and the phenomena of precession and nutation.

Whatever figure and density we suppose in the strata of the Earth, whether or not it be a solid of revolution, provided it differs little from a sphere, we may always assign an elliptic solid of revolution, with which the precession and nutation will be the same. Thus in the hypothesis of Bouguer, of which we have spoken Chap. VII., and according to

which the increase of the degrees varies as the fourth power of the sine of the latitude, these phenomena are exactly the same as if the Earth were an ellipsoid whose ellipticity was $\frac{1}{183}$, but we have seen that observations do not permit us to suppose a greater ellipticity than $\frac{1}{305}$, so that these observations, and the experiments on pendulums, combine to disprove the hypothesis of Bouguer.

We have hitherto supposed the Earth entirely solid, but, this planet being covered in great part by the waters of the ocean, ought not their action to change the phenomena of the precession and nutation? This question it is of importance to consider.

The ocean in consequence of its fluidity is obedient to the action of the Sun and of the Moon. It seems at first sight that their re-action should not affect the axis of the Earth. D'Alembert and every mathematician since, who has investigated these motions, have entirely neglected it, they have even commenced from that

point to reconcile the observed quantit of the precession and nutation, with th measure of the terrestrial degrees. Nevertheless a more profound examinatio of this question has shewn us, that th fluidity of the sea is not a sufficient reason to neglect their effect in the precession of the equinoxes; for if on one hand they obey the action of the Sun and Moon on the other, the force of gravity tends to bring them back without ceasing, to a state of equilibrium, and permits them to make but small oscillations; it is therefore possible, that by their attraction and pression on the spheroid which they cover, they may communicate at least in part, the same motion to the axis of the Earth, which they would if they could possibly become solid. Besides we may by simple reasoning, be convinced that their action is of the same order as the action of the Sun and Moon, on the solid part of the Earth.

Let us imagine this planet homogeneous and of the same density as the ocean, and

moreover that the waters take at every instant the figure that is requisite for the equilibrium of the forces that animate them. If in these hypotheses the Earth should suddenly become entirely fluid, it would preserve the same figure and all its parts would remain in equilibrio, and the axis of the Earth would have no tendency to move, and it is plain that the same should be the case, if a part of this mass should form, by becoming solid the spheroid which the ocean covers. The preceding hypotheses serve as a foundation to the theories of Newton, relative to the figures of the Earth, and of the tides.

It is remarkable, that among the infinite number of those which may be chosen on this subject, this great geometrician has selected two which give neither the precession nor the nutation; the re-action of the waters destroying the effect of the action of the Sun and Moon, upon the terrestrial nucleus, whatever may be its figure. It is true that these

two hypotheses, particularly the last, are not conformable to nature, but we may see, *à priori,* that the effect of the re-action of the waters, although different from that which takes place in the hypothesis of Newton, is nevertheless of the same order.

The investigations which I have made on the oscillations of the ocean, have enabled me to determine this effect of the re-action of the waters in the true hypothesis of nature, and have led to this remarkable theorem.

Whatever may be the law of the depth of the ocean, and whatever the figure of the spheroid which it covers, the phenomena of the precession and nutation will be the same as if the ocean formed a solid mass with this spheroid.

If the Sun and Moon acted only on the Earth, the mean inclination of the equator to the ecliptic would be constant, but we have seen that the action of the planets continually changes the position of the terrestrial orbit: and produces a diminu-

tion of its obliquity to the equator, which is fully confirmed by observations ancient and modern, the same cause gives to the equinoxes a direct annual motion of 0″5707; thus the annual precession produced by the action of the Sun and Moon is diminished by this quantity in consequence of the action of the planets; without this action it would be * 155″20. These effects of the action of the planets are independent of the compression of the terrestrial spheroid, but the action of the Sun and Moon, upon the spheroid, modifies these effects and changes their laws.

If we refer to a fixed plane the position of the orbit of the Earth, and the motion of its axis of rotation, it will appear, that the action of the Sun in consequence of the variations of the ecliptic, will produce in this axis an oscillatory motion similar to the nutation, but with this difference, that the period of these variations being incomparably longer than that of

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* 50″ 3.

the variations of the plane of the lunar orbit, the extent of the corresponding oscillation in the axis of the Earth is much greater than in the nutation. The action of the Moon produces in this same axis a similar oscillation, because the mean inclination of its orbit, to that of the Earth is constant. The displacement of the ecliptic, by being combined with the action of the Sun and Moon upon the Earth, produces upon its obliquity to the equator, a very different variation from that which would arise from this change of position only: the entire extent of this variation would be by this alteration of the ecliptic, about *12 degrees, and the action of the Sun and Moon, reduces it to about †3 degrees.

The variation in the motion of the equinoxes, produced by these same causes, changes the duration of the tropical year in different centuries. The duration diminishes as this motion augments, which is the case

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* 10° 48′. † 2° 42′.

at present, and the actual length of the year is shorter by about *12″, than in the time of Hipparchus. But this variation in the length of the year has its limits, which are restricted by the action of the Sun and Moon, upon the terrestrial spheroid. The extent of these limits would be about † 500″, by the alteration in the position of the ecliptic, but it is reduced to ‡ 120″ by this action.

Lastly, the day itself, such as we have defined it in the First Book, is subject by the displacement of the ecliptic, combined with the action of the Sun and Moon, to very small variations which are indicated by the theory, but are quite insensible to observation. According to this theory the rotation of the Earth is uniform, and the mean length of the day may be supposed constant, an important result for astronomy, as it is the measure of time, and of the revolutions of the heavenly bodies. If it should undergo any change,

* 3″ 8. † 27′. ‡ 38″ 8.

it would be recognized by the alteration in the number of these revolutions, which would be increased or diminished according to their length, but the action of the heavenly bodies does not cause any sensible alteration.

Nevertheless, it might be imagined, that the trade winds which blow constantly from east to west between the tropics, would diminish the velocity of the rotation of the Earth, by their action *on the continents and mountains.* *It is* impossible to submit this action to analysis, fortunately it may be demonstrated that this action on the rotation of the Earth is nothing, by means of the principle of the conservation of areas, which we have explained in the Third Book. According to this principle, the sum of all the particles of the Earth, the ocean and the atmosphere, multiplied respectively by the areas which their radii vectores describe round the centre of gravity of the Earth, projected on the plane

of the equator, is constant in a given time.

The heat of the Sun can produce no effect, because it dilates bodies equally in every direction, and it is evident that if the rotation of the Earth should diminish, this sum would be less. Therefore the trade winds which are produced by the heat of the Sun, cannot alter the rotation of the Earth. To produce any sensible alteration in its period, some great change must take place in the parts of the terrestrial spheroid: thus a great mass taken from the poles to the equator, would make this rotation longer, it would become shorter if the denser materials were to approach the centre or axis of the Earth; but we see no cause that can displace such great masses to such great distances, as to produce any variation in the length of the day, which may be regarded as one of the most constant elements in the system of the world. It is the same with the points where the axis of rotation meets the surface. If the Earth turned round successively different

diameters, making with each other considerable angles, the equator and the poles would change places on the Earth; and the ocean, flowing continually towards the new equator, would alternately overwhelm and then abandon the highest mountains; but all the investigations which I have made upon this change of position in the poles, convince me that it is insensible.

CHAP. XIV.

On the Libration of the Moon.

We have now only to explain the cause of the libration of the Moon, and of the nodes of its equator.

The Moon, in virtue of its motion of rotation, is a little flattened at its poles; but the attraction of the Earth, must have lengthened a little that axis which is turned towards it. If the Moon were homogeneous and fluid, it would (to be in equilibrio) assume the form of an ellipsoid, of which the lesser axis passed through the poles of rotation; the greater axis would be directed to the Earth, and in the plane of the lunar equator, and the mean axis would be situated in the same plane perpendicular to the other two. The excess of the greatest above the least

axis would be quadruple the excess of the mean above the least, and nearly equal $\frac{1}{29711}$, the least axis being taken as unity.

We may easily conceive that if the greater axis of the Moon deviates a little from the direction of the radius vector, which joins its centre with that of the Earth, the terrestrial attraction will tend to bring it down to this radius, in the same manner as gravity brings a pendulum towards the vertical. If the primitive motion of rotation of this satellite, had been sufficiently rapid to have overcome this tendency, the period of its rotation would not have been perfectly equal to that of its revolution, and the difference would have discovered to us successively every point in its surface. But at their origin the angular motions of rotation and revolution having differed but little; the force by which the greater axis of the Moon tended to deviate from the radius vector, was not sufficient to overcome the tendency of this same axis

towards the radius, due to the terrestrial gravity, which by this means has rendered their motions rigorously equal and in the same manner as a pendulum, drawn aside from the vertical by a very small force, continually returns, making small vibrations on each side of it, so the greater axis of the lunar spheroid ought to oscillate on each side of the mean radius vector of its orbit. Hence would arise a motion of libration, of which the extent would depend on the primitive difference between the angular motions of rotation and revolution of the Moon. This difference must have been very small, since it has not been perceived by observation.

Thus we see that the theory of gravitation explains in a sufficiently satisfactory manner, the rigorous equality of these two mean motions of rotation and revolution in the Moon. It would be against all probability to suppose, that these two motions had been at their origin perfectly equal, but for the explanation of this phenomenon, it is enough that their primitive

difference was but small, and then the attraction of the Earth would establish the equality which at present subsists.

The mean motion of the Moon being subject to great secular inequalities, which amount to several circumferences, it is evident that if its mean motion of rotation were perfectly uniform, this satellite would, by virtue of these inequalities, present successively to the Earth every point on its surface, and its apparent disk would change by imperceptible degrees, in proportion as these inequalities were developed; the same observers would see it always pretty nearly the same, and there would be no considerable difference but to observers separated by an interval of several ages. But the cause which has thus established an equality between the mean motions of revolution and rotation, should take away all hope from the inhabitants of the Earth, of seeing the opposite side of the lunar hemisphere. The terrestrial attraction, by continually drawing towards us the

greater axis of the Moon, causes its motion of rotation to participate in the secular inequalities of its motion of revolution, and the same hemisphere to be constantly directed towards the Earth.

The same theory ought to be extended to all the satellites, in which an equality in their motion of rotation and of revolution round their planet has been observed.

The singular phenomenon of the coincidence of the nodes of the equator of the Moon, with those of its orbit, is another consequence of the terrestrial attraction. This was first demonstrated by Lagrange, who by a beautiful analysis was conducted to a complete explanation of all the observed phenomena of the lunar spheroid. The planes of the equator and of the orbit of the Moon, and the plane passing through its centre parallel to the ecliptic, have always very nearly the same intersection; the secular motions of the ecliptic, neither alter the coincidence of the nodes of these three planes, nor their mean inclination,

which the attraction of the Earth constantly maintains the same.

We may observe here, that the preceding phenomena cannot subsist with the hypothesis in which the Moon, originally fluid and formed of strata of different densities, should have taken the figure suited to their equilibrium. They indicate between the axes of the Moon, a greater inequality than would take place in this hypothesis. The great inequalities which we observe at the surface of the Moon, have without doubt a sensible influence on these phenomena.

Whenever nature subjects the mean motions of the celestial bodies to determinate conditions, they are always accompanied by oscillations, whose extent is arbitrary. Thus the equality of the mean motions of revolution and rotation produce a real libration in this satellite. In like manner the coincidence of the mean nodes of the equator and lunar orbit, is accompanied by a libration of the nodes of this equator

round those of the orbit, a libration so small as hitherto to have escaped observation. We have seen that the real libration of the greater lunar axis is insensible, and *we have observed*, (Chap. VI.) *that* the libration of the three satellites of Jupiter is also insensible. It is remarkable, that these librations, whose extent is arbitrary and might have been considerable, should nevertheless be so very small; we must attribute this to the same causes as originally established the conditions on which they depend.

But relatively to the arbitrary quantities, which relate to the initial motion of rotation of the celestial bodies, it is natural to think that without foreign attractions, all their parts, in consequence of the friction and resistance which is opposed to their reciprocal motion, would in process of time, acquire a permanent state of equilibrium, which cannot exist but with an uniform motion of rotation, round an invariable axis; so that observation should no longer indicate in this motion any other

inequalities than those derived from these attractions. The most exact observations shew that this is the case with the Earth, the same result extends to the Moon, and probably to the other celestial bodies.

CHAP. XV.

Reflections on the Law of Universal Gravitation.

In considering the whole of the phenomena of the solar system, we may arrange them in the three following classes:

The first embraces the motions of the centre of gravity about the foci of the principal forces which animate them.

The second includes all that relates to the figure and oscillations of the fluids that surround them.

And the third comprehends the motions of these bodies round their centres of gravity. It is in this order that we have explained the different phenomena, and we have seen that they are necessary consequences of the principle of gravitation. This principle has made us acquainted with a great number of inequalities, which

it would have been impossible to have unravelled by observation alone; it has furnished us the means of subjecting the heavenly motions to sure and precise rules. The astronomical tables, founded only on the theory of gravitation, borrow now from observation, only such arbitrary quantities as cannot otherwise be known, and we can only hope to add to their perfection by giving greater precision, both to our observations and our theory.

The motion of the Earth, which had obtained the assent of astronomers, from the simplicity with which it explained the celestial phenomena, has received from the principle of gravitation a new confirmation, which has carried it to the highest degree of evidence of which physical science is susceptible. We may increase the probability of a theory, either by diminishing the number of hypotheses on which it rests, or by augmenting the number of phenomena which it explains. The principle of gravity has procured these two advantages to the theory of the mo-

tion of Earth. As it is a necessary consequence of it, it adds no new supposition to this theory; but to explain the apparent motion of the stars, Copernicus admitted three distinct motions, one round the Sun, another round itself, and a third motion of its poles round those of the ecliptic. The principle of gravitation makes them all depend on one motion impressed on the Earth, in a direction not passing through the centre of gravity. In consequence of this motion, it revolves round the Sun, and on its own axis, it at the same time takes a flattened form, compressed at the poles, and the action of the Sun and Moon upon this figure, produces a slow motion on its poles, round the poles of the ecliptic. The discovery of this principle has then reduced to the least possible number, the suppositions on which Copernicus founded his theory. It has besides the advantage of connecting this theory with all the celestial phenomena. Without it, the ellipticity of the planetary orbits, the laws which the planets and

comets follow in their revolution round the Sun, their secular and periodic inequalities, the numberless inequalities of the Moon, and of the satellites of Jupiter, the precession of the equinoxes, the nutation of the terrestrial axis, the motions of the lunar axis, and lastly, the ebbing and flowing of the sea, would only be insulated and unconnected phenomena. It is really a circumstance deserving our admiration, the manner in which all these phenomena, at first sight so unconnected, flow from one law which connects them with the motion of the Earth; so that, this motion once admitted, we are conducted by a series of geometrical reasoning to these phenomena. Each of them furnishes, therefore, a proof of its existence, and if we consider that there does not exist a single phenomena which cannot be referred to the law of gravity, and that this law determines with the greatest exactness the positions and motions of the heavenly bodies through the whole of their course, there will be no reason to fear that its

truth will be questioned, in consequence of any phenomena hitherto unobserved; and finally, when we see that Uranus with its satellites lately discovered, obey and confirm the same law, it is impossible to refuse assent to these proofs, and not to allow that nothing in natural philosophy is more completely demonstrated than the motion of the Earth, and the principle of universal gravitation, in proportion to the masses, and inversely as the squares of the distances.

Is this principle a primordial law of nature? Or is it a general effect of an unknown cause? Here we are stopped by our ignorance of the nature of the intimate properties of matter, and deprived of every hope of answering this question in a satisfactory manner. Instead of forming hypotheses on this subject, let us content ourselves with examining more particularly the manner in which this principle has been employed by philosophers.

They have admitted the following five suppositions:

1. That gravitation takes place between the most minute particles of bodies.

2. That it is proportional to their masses.

3. That it is inversely as the squares of the distances.

4. That it is transmitted instantaneously from one body to another.

5. And that it equally acts on bodies in a state of repose, and upon those which, moving in its direction, seem in part to withdraw themselves from its activity.

The first of these suppositions is, as we have seen, a necessary result of the equality which exists between action and reaction; every particle of the Earth attracting it, as the particle itself is attracted. This supposition is moreover confirmed by the measures of the degrees of the meridian, and by experiments on pendulums; for amidst all the irregularities of the measured degrees, we may per-

ceive the traces of regular figure, which is conformable to the theory. The great influence the compression of Jupiter, has upon *the nodes* and *perijoves of the* orbits of its satellites, proves to us that the attraction of this planet is composed of the attractions of all its particles.

The proportionality *of the* attractive force to the masses, is demonstrated on the Earth by experiments on pendulums, the oscillations of which are of the same length of whatever substance they are composed. It is proved in the celestial regions, by the constant relation which exists between the squares of the periodic times of bodies, revolving about a common focus, to the cubes of the greater axis of their orbits.

We have seen in the First Chapter with what precision the almost absolute state of repose of the perihelia of the planetary orbits, indicate that the force of gravity varies according to the inverse square of the distance, and now that we know the cause of the motions of these perihelia,

we may regard this law as rigorously exact. It is the same with all emanations which proceed from a centre, such as light; it seems as if all forces whose action is perceived at sensible distances follow this law. It has lately been observed, that the attractions and repulsions of electricity and magnetism decrease in proportion to the squares of the distances. A remarkable property of this law is, that if the dimensions of all the bodies of the universe, their mutual distances and velocities, were to be augmented or diminished proportionally, they would describe curves entirely similar to those described at present, and their appearances would be entirely the same. For the forces which animate them, being the result of attractions, proportional to the masses divided by the squares of the distances, they would augment and diminish proportionally as the dimensions of this imaginary universe. It may be remarked at the same time that this property can only belong to the law of nature. Thus the appearances of the

motions of the universe, are independent of its absolute dimensions, as they are likewise of the absolute motion it may have in space, and we can only observe and recognize relative phenomena.

It is this law which gives to spheres the property of attracting each other mutually, as if their whole masses, were united at their respective centres. It terminates also the orbits and the figures of the celestial bodies, by lines and surfaces of the second order, at least if we neglect their perturbations and suppose them fluid.

We have no method of measuring the length of time in which gravity is propagated, because the action of the Sun having once attained the planets, it continues to act on them as if the attractive force was communicated instantaneously to the extremities of the system; we cannot therefore ascertain in how long a time it is transmitted to the Earth, no more than we could measure the velocity of light, were it not for the aberration and

eclipses of Jupiter's satellites. But it is not the same with the small difference that may exist in the action of gravity upon bodies, according to the direction and quantity of their velocity. Analysis has shewn me, that there should result an acceleration in the mean motions of the planets round the Sun, and in the mean motions of the satellites about their planets.

I had imagined this method of explaining the secular equation of the Moon, when I believed with other geometricians that it was inexplicable on the principle of universal gravitation. I found that if it arose from this cause, we must suppose in the Moon, in order to release it entirely from its gravity towards the Earth, a velocity in the centre of this planet, at least six million times greater than that of light; the true cause of this equation being now known, we are certain that the activity of gravity is much greater than this. This force therefore acts with a velocity which we may consider as infinite, and we may

of the integrant particles? To admit this hypothesis we must suppose much more space empty than full in all bodies, so that the density of their particles must be incomparably greater than the mean density of their whole volume. A spherical particle of one hundred thousandth of a foot in diameter, should have a density at least ten thousand millions of times greater than the mean density of the Earth, to exert at its surface an attraction equal to the terrestial gravity. But the attractive forces of bodies greatly surpass this gravity, since they inflect light, whose direction is not sensibly changed by the attraction of the Earth; the density of these particles therefore should be to that of bodies in a ratio, which the imagination would fear to admit, if their affinities depended on the law of universal gravitation. The ratio of the intervals, which separate the particles of bodies, to their respective dimensions, would be of the same order, as relatively to stars which form a nebula, which in this point of view may be consi-

dered as a great luminous body. There is no reason, however, which absolutely forbids us to consider all bodies in this manner. Many phenomena are favourable to the supposition, particularly the extreme facility with which light penetrates diaphonous bodies in all directions. The affinities would then depend on the form of the integrant particles, and we might then by the variety of these forms, explain all the variety of attractive forces, and reduce to one general law all the phenomena of astronomy and natural philosophy.

But the impossibility of ascertaining these figures, renders this investigation useless to the advancement of science. Some geometricians, to account for these affinities, have added to the laws of attraction, inversely as the squares of the distance, new terms which are insensible at small distances, but these terms would be the expressions of as many different forces, and besides being complicated with the figures of the particles, they would only complicate the explanation of the phenomena.

Amidst these uncertainties the wisest plan seems to be, to endeavour to determine by numberless experiments the laws of affinities, and to effect this, the most simple method appears to be, by comparing these forces with the repulsive force of heat, which may itself be compared with that of gravity. Some experiments already *made with this* view, afford us reason to hope that one day these laws will be perfectly known, and that then, by the application of analysis, the philosophy of terrestrial bodies may be brought to the same degree of perfection, which the discovery of universal gravitation has procured to astronomy.

THE

SYSTEM OF THE WORLD.

BOOK V.

Of the History of Astronomy.

The order in which I have treated the principal results of the system of the world, is not that which the human intellect has followed in the investigation. Its progress has been embarrassed and uncertain. Frequently mankind have not arrived at the true cause of these phenomena, till all the hypotheses which imagination could suggest have been exhausted; and the truths that have been discovered, have almost always been combined with errors, which time and observation only have separated. I shall comprise in a small compass, an outline of these attempts

and their success. We shall see that astronomy remained during a great many ages in its infancy, that it increased and flourished in the school of Alexandria, became then stationary till the time of the Arabs, who improved it by their observations, and lastly that it is within the three last centuries, it has rapidly risen to that state of perfection, in which we behold it at the present day.

CHAP. I.

Of the Astronomy of the Ancients; till the Foundation of the Alexandrine School.

THE view of the firmament must at all times have fixed the attention of mankind, and more particularly in those happy climates, where the serenity of the air invited them to observe the stars. Agriculture required that the seasons should be distinguished and their returns known. It could not be long before it was discovered that the rising and setting of the stars, when they plunge themselves in the Sun's rays, or when they again disengage themselves from his light, might answer this purpose. Hence we find that among most nations, this species of observations may be traced back to such early times, till their origin is lost. But

some rude remarks on the rising and setting of the stars, could not constitute a science. Astronomy did not commence till observations being registered and compared, and the celestial motions examined with greater care, some attempt was made to explain their motions and their laws.

The motion of the Sun in an orbit inclined to the equator; the motion of the Moon, its phases and eclipses, the knowledge of the planets and their revolutions, and the sphericity of the Earth, were probably the objects of this ancient astronomy, but the few monuments that remain of it are insufficient to ascertain either its epoch or its extent. We can only judge of its great antiquity, by the astronomical periods which it has transmitted to us, by some just notions which the Egyptians and Chaldeans seem to have had of the system of the world, and by the exact relation of the ancient measures to the circumference of the Earth. Such has been the vicissitude of human affairs, that the

arts by which alone the events of past ages can be transmitted in a durable manner, being of modern invention, the remembrance of the first inventors in the arts and sciences, has been entirely effaced. Great nations, whose names are hardly known in history, have disappeared from the soil which they inhabited; their annals, their language, and even their cities have been obliterated, and no remnant left of their science or their industry, but a confused tradition, and some scattered ruins, of doubtful and uncertain origin.

It appears that the practical astronomy of these early ages, was confined to the observations of eclipses, the rising and setting of the principal stars, with their occultations by the Moon and planets. The path of the Sun was followed by means of the stars which were eclipsed by the twilights, and perhaps by the variations in the meridian shadow of the gnomon. The motion of the planets was determined by the stars which they came nearest to, in their course. To distinguish

these bodies, and recognize their various motions, the heaven was divided into constellations. And that zone from which the Sun, Moon and planets were never seen to deviate, was called the zodiac. It was divided into the twelve following constellations. Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus and Pisces. These were called *signs*, because they served to distinguish the seasons. Thus the entrance of the Sun into Aries, in the time of Hipparchus, marked the commencement of the spring, after which it described the other signs Taurus, Gemini, &c. but the retrograde motion of the equinoxes, changed the coincidence of the seasons; nevertheless, observers accustomed to mark the commencement of the spring by the entrance of the Sun into the sign of Aries, have continued to mark this in the same manner, and have distinguished the signs of the zodiac from the constellations, the first being ideal, and serving only to designate the course of the Sun in the ecliptic. Now that we endeavour to refer

our ideas to the most simple expressions, we begin no longer to use the signs of the zodiac, but mark the positions of the heavenly bodies on the ecliptic, according to their distance from the equinoctial point.

Some of the names given to the constellations of the zodiac, appear to relate to the motion of the Sun. Cancer, for example, seems to indicate the retrogradation of this body from the solstice, and the balance denotes the equality of day and night. And other names seem to refer to the climate and agriculture of those nations to whom the zodiac owes its origin. The most ancient observations that have been transmitted to us with sufficient detail, are three eclipses of the Moon, observed at Babylon in the years 719 and 720 before the Christian æra. Ptolemy, who cites them in his Almagist, employs them in his determination of the motion of the Moon. It is certain, that neither he nor Hipparchus could obtain any that were more ancient, for the exactness of the comparison is in proportion to the

interval which separates the extreme observations. This consideration should diminish our regret for the loss of nineteen hundred years of observations by the Chaldeans, and of which they boasted in the time of Alexander, and which Aristotle obtained by means of Calysthenes. But they could only have discovered the period of 6585 days, by a long series of observations. This period, called the σαρος, has the advantage of bringing back the Moon to nearly the same period, with respect to its node, its perigee, and to the Sun. Thus, the eclipses observed in one period, afford an easy method of calculating those which are to happen in the succeeding ones. The lunar-solar period of six hundred years, seems to have been known to the Chaldeans. These two periods suppose a knowledge nearly approximating to the true length of the year; it is also highly probable, that they had remarked the difference between the sidereal and tropical year, and that they were acquainted with the use of the gnomon and

sun-dial. And finally, some of them were led from considering the spectacle of nature, to suppose that comets, like planets, are subject to fixed periods, which are regulated by external laws.

Astronomy is not less ancient in Egypt than in Chaldea. The Egyptians were acquainted, long before the christian æra, with the excess of the year, of one quarter of a day beyond 365 days: on this knowledge, they formed the sothic period of 1460 years, which, according to them, brought back the same seasons, months, and festivals of their years, whose length was 365 days. The exact direction of the sides of their pyramids with the four cardinal points, give us a very advantageous idea of their accuracy of observation. It is probable, that they had also methods of calculating eclipses. But that which reflects most honour to their astronomy, was the sagacious and important observation of the motion of Mercury and Venus about the Sun. The reputation of their priests attracted to them the greatest

philosophers of Greece; and, according to all appearance, the school of Pythagoras is indebted to them for the sound notions they professed, relative to the system of the universe.

Among these people, astronomy was only cultivated in their temples, and by priests, who made no other use of their knowledge, than to consolidate the empire of superstition, of which they were the ministers. They carefully disguised it under emblems, which presented to credulous ignorance, heroes and gods, whose actions were only allegories of celestial phenomena, and of the operations of nature; allegories which the power of imitation, one of the chief springs of the moral world, has perpetuated to our own days, and been mingled with our religious institutions. The better to enslave the people, they profited by their natural desire of penetrating into futurity, and created astrology. Man being induced, by the illusions of his senses, to consider himself as the centre of the universe, it

was easy to persuade him, that the stars influenced the events of his life, and could prognosticate to him his future destiny. This error, dear to his self-love, and necessary to his restless curiosity, seems to have been co-eval with astronomy. It has maintained itself through a very long period, and it is only since the end of the last century, that our knowledge of our true relations with nature, has caused them to disappear. In Persia and in India, the commencement of astronomy is lost in the darkness which envelopes the origin of these people. In no country do they go back so far as in China, by an incontestable series of historical monuments.

The prediction of eclipses, and the regulation of the calendar, were always regarded as important objects, for which a mathematical tribunal was established; but the scrupulous attachment of the Chinese, for their ancient customs, which extended even to their astronomical rules, has contributed with them, to keep this science in a perpetual state of infancy.

The Indian tables indicate a much more refined astronomy, but every thing shews that it is not of an extremely remote antiquity. And here, with regret, I differ in opinion from a learned and illustrious astronomer, who, after having honoured his career by labours useful both to science and humanity, perished a victim to the most sanguinary tyranny, opposing the calmness and dignity of virtue, to the revilings of an infatuated people, who wantonly prolonged the last agonies of his existence.

The Indian tables have two principal epochs, which go back, one to the year 3102, the other to the year 1491 before the Christian æra. These epochs are connected with the mean motions of the Sun, Moon, and planets, in such a manner, that one is evidently fictitious; the celebrated astronomer, above alluded to, endeavours, in his Indian astronomy, to prove, that the first of these epochs is founded on observation. Notwithstanding, all the arguments are brought forward

with that interest he so well knew how to bestow on subjects the most difficult, I am still of opinion, that this period was invented for the purpose of giving a common origin to all the motions of the heavenly bodies in the zodiac. In fact, computing, according to the Indian tables, from the year 1491, to 3102, we find a general conjunction of the Sun and all the planets, as these tables suppose, but their conjunction differs too much from the result of our best tables, to have ever taken place, which shews that the epoch to which they refer, was not established on observation. But, it must be owned, that some elements of the Indian astronomy, seem to indicate that they have been determined even before this first epoch. Thus the equation of the centre of the Sun, which they fix at 2°.4173, could not have been of that magnitude; but at the year 4300 before the Christian æra. But, independently of the errors to which the Indian observations are liable, it may be observed, that they only

considered the inequalities of the Sun and Moon, relative to eclipses, in which the annual equation of the Moon is added to the equation of the centre of the Sun, and augments it about *22′, which is very nearly the difference between our determinations, and those of the Indians. Many elements, such as the equations of the centre of Jupiter and Mars, are so different in the Indian tables, from what they must have been at their first epoch, that we can conclude nothing in favour of their antiquity, from the other elements.

The whole of these tables, particularly the impossibility of the conjunction, at the epoch they suppose, prove, on the contrary, that they have been constructed, or at least rectified in modern times. Nevertheless, the ancient reputation of the Indians does not permit us to doubt, but that they have always cultivated astronomy, and the remarkable exactness of the mean motions which they have as-

* 11′ 52″.

signed to the Sun and Moon, necessarily required very ancient observations.

The Greeks did not begin to cultivate astronomy, till a long time after the Egyptians, of whom they were the disciples.

It is extremely difficult to ascertain the exact state of their astronomical knowledge, amidst the variety of fable which fills the early part of their history. It appears, however, that they divided the heavens into constellations, about thirteen or fourteen centuries before the Christian æra; for it is to this epoch that the sphere of Eudoxus should be referred. Their numberless schools for philosophy, produced not one single observer, before the foundation of the Alexandrine school. They treated astronomy as a science purely speculative, often indulging in the most frivolous conjectures.

It is singular, that at the sight of so many contending systems, which taught nothing, the simple reflection, that the only method of comprehending nature, is to interrogate her by experiment, never occurred to one of these philosophers,

though so many were endowed with an admirable genius. But we must reflect, that the first observation only presenting insulated facts, little suited to attract the imagination, impatient to ascend to causes, they must have succeeded each with extreme slowness. It required a long succession of ages to accumulate a sufficient number, to discover, among the various phenomena, such relations which by extending themselves should unite with the interest of truth, that of such general speculations as the human understanding delights to indulge in.

Nevertheless, in the philosophic dreams of Greece, we trace some sound ideas, which their astronomers collected in their travels, and afterwards improved. Thales born at Miletus, 640 years before our æra, went to Egypt for instruction: on his return to Greece, he founded the Ionian school, and there taught the sphericity of the Earth, the obliquity of the ecliptic, and the true causes of the eclipses of the Sun and Moon; he even went so far as to

riods which had been communicated to him by the priests of Egypt.

Thales had for his successors—Anaximander, Anaximenes, and Anaxagoras; to the first is attributed the invention of the gnomon and geographical charts, which the Egyptians appear to have been already acquainted with.

Anaxagoras was persecuted by the Athenians for having taught these truths of the Ionian school. They reproached him with having destroyed the influence of the gods on nature, by endeavouring to reduce phenomena to immutable laws. Proscribed with his children, he only owed his life to the protection of Pericles, his disciple and his friend, who succeeded in procuring a mitigation of his sentence, from death to banishment. Thus, *truth*, to establish itself on earth, has almost always had to combat established prejudices, and has more than once been fatal to those who have discovered it. From the Ionian school arose the chief of one more celebrated. Pythagoras, born at Samos, about

590 years before Christ, was at first the disciple of Thales. This philosopher advised him to travel into Egypt, where he consented to be initiated into the mysteries of the priests, that he might obtain a knowledge of all their doctrines. The Brachmans having then attracted his curiosity, he went to visit them, as far as the shores of the Ganges. On his return to his own country, the despotism under which it groaned, obliged him again to quit it, and he retired to Italy, where he founded his school. All the astronomical truths of the Ionian-school, were taught on a more extended scale in that of Pythagoras; but what principally distinguished it, was the knowledge of the two motions of the Earth on itself, and about the Sun. Pythagoras carefully concealed this from the vulgar, in imitation of the Egyptian priests, from whom, most probably, he derived his knowledge; but his system was more fully explained, and more openly avowed by his disciple Philalaus.

According to the Pythagoricians, not

only the planets, but the comets themselves, are in motion round the Sun. These are not fleeting meteors formed in the atmosphere, but the eternal works of nature. These opinions, so perfectly correct on the system of the universe, have been admitted and inculcated by Seneca, with the enthusiasm which a great idea, on the subject the most vast of human contemplation, naturally excited in the soul of a philosopher.

"Let us not wonder," says he, "that we are still ignorant of the law of the motion of comets, whose appearance is so rare, that we neither can tell the beginning nor the end of the revolution of these bodies, which descend to us from an immense distance. It is not fifteen hundred years, since the stars have been numbered in Greece, and names given to the constellations. The day will come, when, by the continued study of successive ages, things which are now hid, will appear with certainty, and posterity will wonder they have escaped our notice."

In the same school, they taught that the planets were inhabited, and that the stars were suns, disseminated in space, being themselves centres of planetary systems. These philosophic views should, from their grandeur and justness, have obtained the suffrages of antiquity; but having been taught with systematic opinions, such as the harmony of the heavenly spheres, and wanting, moreover, that proof which has since been obtained, by the agreement with observations, it is not surprising that their truth, when opposed to the illusions of the senses, should not have been admitted.

CHAP. II.

Of Astronomy, from the Foundation of the Alexandrine School, to the Time of the Arabs.

Hitherto, the practical astronomy of different people, has only offered us some rude observations relative to the seasons and eclipses; objects of their necessities or their terrors. Their theoretical astronomy consisted in the knowledge of some periods, founded on very long intervals of time, and of some fortunate conjectures, relative to the constitution of the universe, but mixed with considerable error. We see, for the first time, in the school of Alexandria, a connected series of observations; angular distances were made with instruments suitable to the purpose, and they were calculated by trigometrical methods. Astronomy then took a new

form, which the following ages have adopted and brought to perfection. The positions of the fixed stars were determined, the paths of the planets carefully traced, the inequalities of the Sun and Moon were better known, and, finally, it was the school of Alexandria that gave birth to the first system of astronomy, that had ever comprehended an entire plan of the celestial motions. This system was, it must be allowed, very inferior to that of the school of Pythagoras, but being founded on a comparison of observations, it afforded, by this very comparison, the means of its own destruction, and the true system of nature has been elevated on its ruins.

After the death of Alexander, his principal generals divided his empire among themselves, and Ptolemy Soter received Egypt for his share. His munificence and love of the sciences, attracted to Alexandria, the capital of his kingdom, a great number of the most learned men of Greece. Ptolemy Philadelphus, who inherited with the kingdom his father's love of the sci-

ences, established them there under his own particular protection. A vast edifice, in which they were lodged, contained both an observatory, and that magnificent library which Demetrius Phalerus had collected with such trouble and expence. Here they were supplied with whatever books and instruments were necessary to their pursuits; and their emulation was excited by the presence of a prince, who often came amongst them to participate in their conversation and their labours.

Arystillus and Thimocares were the first observers of this rising school; they flourished about the year 300 before the Christian æra. Their *observations of the* principal stars of the zodiac, enabled Hipparchus to discover the precession of the equinoxes, and Ptolemy, from their observations of the planets, founded his theory of those bodies.

The next astronomer which the school of Alexandria produced, was Aristarchus, of Samos. The most delicate elements of astronomy, were the subjects of his inves-

tigation. He observed the summer solstice, the year 281 before the Christian æra. He determined the magnitude of the apparent diameter of the Sun, which he found equal to the 720th part of the whole circumference, which quantity is a mean between the two limits, which Archimedes assigned, a few years afterwards, to this diameter, by an ingenious method, according to which the solar diameter appeared to him greater than the 200th part of a right angle, and less than the 164th part. But that which reflects the greatest honour on the genius of Aristarchus, is the method by which he endeavoured to determine the distance of the Sun from the Earth. He observed the angle contained between the Sun and the Moon, at the moment he judged half of the lunar disk to be illuminated by the Sun, and having found it just 96°.7, he concluded that the Sun was eighteen or twenty times farther from us, than the Moon. Notwithstanding the inaccuracy of this result, it extended the boundaries of the universe much farther than had

been done before. Aristarchus revived the opinion of the Pythagoricians, relative to the motion of the Earth. But as his writings have not been transmitted to us, we are ignorant to what extent he carried this theory in his explanation of the celestial phenomena. We only know that this judicious astronomer, having reflected that the motion of the Earth produced no change in the apparent position of the stars, placed them at a distance incomparably greater than the Sun. Thus it appears, that of all the ancient astronomers, Aristarchus had formed the most just notions of the magnitude of the universe.

The celebrity of his successor, Eratosthenes, is principally due to his measure of the Earth, and his observations on the obliquity of the ecliptic. Having, at the summer solstice, remarked a deep well, whose whole depth, was illuminated by the Sun, at Syene, in Upper Egypt, he compared this with the altitude of the Sun, observed at the same solstice at Alexandria. He found the celestial arc, contained be-

tween the zeniths of these two places, equal to the 50th part of the whole circumference, and as their distance was estimated at 500 stadia, he fixed at 250 thousand stadia, the length of the whole terrestrial circumference. The uncertainty that exists, as to the value of this stadium, does not permit us to appreciate the exactness of this measurement.

Aristotle, Cleomedes, Possidonius, and Ptolemy, have given four other evaluations of the circumference of the Earth, equivalent to 400, 300, 240, 180 thousand stadia. The simple relation of these measures to each other, leave room to conjecture, that these different quantities are translations of the same measure, in different stadia. The Alexandrian stadium was 400 great cubits, of the same length as the nilometer of Cairo, which, according to Freret, has not been altered for a great number of centuries, and may be traced back to the time of Sesostris; its magnitude is equal to 1.7119 feet, according to some measures lately made with

great precision, which gives 684,76 feet, for the value of the stadium of Alexandria. As it is probable this stadium was that of Ptolemy, the circumference of the Earth, according to that astronomer, would be 123256800 feet, which differs but little from our actual measurement, which fixes it at 123178320 feet.*

If the measures of Possidonius, Cleomedes, and Aristotle, are identical with that of Ptolemy, the corresponding stadia are 513,570, 410,856, and 308,142 feet. Now, in comparing a great number of ancient itinerary distances with the actual known distances, we find in antiquity these different stadia so precisely, as to render the identity of these four measures of the Earth extremely probable. It is therefore very probable, that they all depend on some ancient and very exact measure, either executed with great care, or in which the errors were fortunately compensated, as has since happened in the measure of a

* Ten millions of metres, according to the new measurement.

degree, by Fernel, and even in that by Picard. It is true, we know, that Possidonius himself measured an arc of the terrestrial meridian; and his operation, as far as we can judge from the details that have been transmitted to us, was very inexact; but there is reason to think he only proposed to verify some ancient measures of the Earth, and that he found them to agree nearly with his own.

The observation of Eratosthenes, on the obliquity of the ecliptic, is very valuable, inasmuch as it confirms the diminution of it, determined *à priori*, by the theory of gravitation. He found the distance between the tropics less than 53.06, and greater than 52.96, which gives us a mean * 26.50, for the obliquity of the ecliptic. Hipparchus found no reason to alter this result by his observations.

But, of all the astronomers of antiquity, the science is most indebted to Hipparchus of Bythinia, for the great number and extent of his observations, by the important results he obtained, by comparing them

* 23.51.

with those that had been formerly made by others; and for the excellent method which he pursued in his researches. He flourished at Alexandria about 140 years before our æra. Not content with what had already been done, he determined to recommence every thing, and not to admit any results but those founded on a new examination of former observations, or on new observations, more exact than those of his predecessors.

Nothing affords a stronger proof of the uncertainty of the Egyptian and Chaldean observations on the Sun and stars, than the necessity which compelled him to recur to the observations of the Alexandrine school, to establish his theories of the Sun, and of the precession of the equinoxes. He determined the length of the tropical year, by comparing one of his observations of the summer solstice, with one made by Aristarchus of Samos, forty-five years before; he found it 365,24667 days. This is in excess about four minutes and a half. But he remarks himself on the little reliance that can be placed on a determina-

tion from solstitial observations, and on the advantage of employing observations of the equinoxes. Hipparchus recognized that there elapsed 187 days from the vernal equinox, to that of the autumn, and 178 days only from this last equinox, to that of the spring. He observed, likewise, that these intervals were unequally divided by the solstices, so that 94 days and a half elapse from the vernal equinox to the summer solstice, and 92 days and a half from this solstice to the autumnal equinox.

To explain these differences, Hipparchus supposed the Sun to move uniformly in a circular orbit; but, instead of placing the Earth in the centre of it, he supposed it removed the 24th part of the radius, and fixed the apogee at the sixth degree of Gemini. From these data he formed the first solar tables to be found in the History of Astronomy. The equation of the centre, which they suppose was too great, it is very probable, that a comparison with eclipses, in which this equation is augmented by the annual equation of the Moon, confirmed Hipparchus in his

error, and perhaps even led him into it. He was mistaken also in supposing circular the elliptic orbit of the Sun, and that the real velocity of this body was constantly uniform. The contrary is now demonstrated by direct measures of the Sun's apparent diameter; but such observations were impossible at the time of Hipparchus, whose solar tables, with all their imperfections, are a lasting monument of his genius, which Ptolemy, three centuries after, respected, but did not attempt to improve.

This great astronomer next considered the motions of the Moon; he measured the length of its revolution by comparing eclipses, and determined both the excentricity and inclination of its orbit, he ascertained the motion of its nodes and of its apogee, and from the determination of its parallax endeavoured to conclude that of the Sun, by the breadth of the cone of the terrestrial shadow, in an eclipse at the moment it was traversed by the Moon, which led him nearly to the same result

as had been obtained by Aristarchus. He made a great number of observations on the planets, but too much the friend of truth to explain their motions by uncertain theories, he left the task of this investigation to his successors. A new star which appeared in his time induced him to undertake a catalogue of the fixed stars, to enable posterity to recognize any changes that might take place in the appearances of the heavens. He was sensible also of the importance of such a catalogue for the observations of the Moon and the planets. The method he employed was that of Arystillus and Timochares, and which we have already explained in the First Book. The reward of this long and laborious task was the important discovery of the precession of the equinoxes; in comparing his observations with those of these astronomers, he discovered that the stars had changed their situation with respect to the equator, but had preserved the same latitude with respect to the ecliptic, so that to explain these different

changes, it is sufficient to give a direct motion to the celestial sphere round the poles of the ecliptic, which produces a retrograde motion of the equinoxes with respect to the stars. But he announced his discovery with some reserve, being doubtful of the accuracy of the observations of Arystillus and Timochares. Geography is indebted to Hipparchus for the method of determining places on the Earth, by their latitude and longitude, for which he first employed the eclipses of the Moon. Spherical trigonometry, also, owes its origin to Hipparchus, who applied it to the numberless calculations which these investigations required. His principal works have not been transmitted to us; they perished in the conflagration of the Alexandrine library, and we are only acquainted with them through the Almagest of Ptolemy.

The interval of near three centuries which separated these two astronomers, produced some observers, as Agrippa, Menelaus, and Theon. We may also notice in this inter-

val the reformation of the calendar by Julius Cæsar, and the precise knowledge of the ebbing and flowing of the sea. Possidonius observed the law of this phenomenon, which appertains to astronomy by its evident relation to the motion of the Sun and Moon, and of which Pliny the naturalist has given a description remarkable for its exactness.

Ptolemy, born at Ptolemais in Egypt, flourished at Alexandria, about the year 130 of our æra. Hipparchus had conceived the project of reforming astronomy, and establishing the science on new foundations. Ptolemy continued this labour, too vast to be accomplished by a single individual, and has given a complete treatise on this science in his great work entitled the Almagest.

His most important discovery is the evection of the Moon. Astronomers previously had only considered the motion of this body relative to eclipses; by following it through its whole course, Ptolemy recognized, that the equation of the centre

of the lunar orbit, was less in the syzigies than in the quadratures; he determined the law of this difference, and ascertained its value with great precision. To represent it, he made the Moon to move upon an epicycle carried by an excentric, according to a method attributed to Appollonius the geometrician, and which had before been employed by Hipparchus.

It was a general opinion of the ancients, that the uniform circular motion being the most simple and natural, was necessarily that of the heavenly bodies. This error maintained its ground till the time of Kepler, and for a long time impeded him in his researches. Ptolemy adopted it, and, placing the Earth on the centre of the celestial motions, he endeavoured to represent their inequalities in this false hypothesis. Eudoxus had previously imagined for this object, every planet attached to several concentric spheres, endowed with different motions; but this astronomer not having explained in what manner these spheres, by their action on

the planets produce the variety of their motions. His hypothesis hardly deserves notice, in a treatise on astronomy. A much more ingenious hypothesis consists in moving along one circumference, of which the Earth occupies the centre, that of another circumference, on which moves that of a third, and so on, up to the last circumference, on which the body is supposed to move uniformly. If the radius of one of these circles surpasses the sum of the others, the apparent motion of the body round the Earth, will be composed of a mean uniform motion, and of several inequalities depending on the proportions of these several radii to each other, and the motions of their centres, and of that of the Star. By increasing their number, and giving them suitable dimensions, we may represent the inequalities of this apparent motion. Such is the most general manner of considering the hypothesis of cycles and excentrics, which Ptolemy adopted in his theories of the Sun, Moon, and planets. He supposed these bodies

in motion round the Earth in this order of distances—the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn; astronomers were divided in their opinions as to the position of Mercury and Venus; Ptolemy followed the most ancient opinion, and placed them below the Sun; others placed them above, and finally, the Egyptians made them move round it. It is singular, that Ptolemy does not mention this hypothesis, which is equivalent to placing the Sun in the centre of the epicycles of these two planets, instead of making them revolve round an imaginary centre. But, being persuaded that his system could only be adapted to the three superior planets, he transferred it to the two inferior, and was misled by a false application of the principle of the uniformity of the laws of nature, which, if he had set out from the discovery of the Egyptians, on the motions of Mercury and Venus, would have led him to the true system of the world. But even, if epicycles could be made to represent the ine-

qualities of the motions of the heavenly bodies, still it would be impossible to represent the variations in their distances. In the time of Ptolemy, these variations were almost insensible in the planets, whose apparent diameters could not then be measured. But his observations on the Moon should have taught him that his hypothesis was erroneous, according to which the diameter of the Moon perijee, in the quadratures, should be double of the diameter apogee in the sysigies. The motion in latitude of the planets, was another difficulty to be unexplained by this system; and every inequality which the improvements in the art of observing discovered, incumbered this system with a new epicycle, which, instead of being confirmed by the progress of the science, has only grown more and more complicated; and this should convince us, that it is not that of nature. But in considering it as a method of adapting the celestial motions to calculation, this first attempt of the human understanding towards

an object so very complicated, does great honour to the sagacity of its author.

Ptolemy confirmed the motion of the equinoxes, discovered by Hipparchus, by comparing his observations with those of this great astronomer. He established the respective immobility of the Stars, their invariable latitude to the ecliptic, and their motion in longitude, which he found * 111″ in every year, as Hipparchus had suspected.

We now know, that this motion is very nearly † 154″ annually, which, considering the interval between the observations of Ptolemy and Hipparchus, implies an error of more than one degree in their observations. Notwithstanding the difficulty which attended the determination of the longitude of the Stars, when observers had no exact measure of time, we are surprised that so great an error should have been committed, particularly when we observe the agreement of the observations with each other, which Ptolemy cites as

* 35″.9. † 50″.

a proof of the accuracy of his result. He has been reproached with having altered them, but this reproach is not founded; his error, in the determination of the motion of the equinoxes, seems to have been derived from too great confidence in the result of Hipparchus, relative to the length of the tropical year and the motion of the Sun. In fact, Ptolemy determined the longitudes of the stars, by comparing them either with the Sun, or with the Moon, which was equivalent to a comparison with the Sun, since the synodical revolution of the Moon was well known by the means of eclipses. Now, Hipparchus having supposed the year too long, and consequently the motion of the Sun in longitude too slow, it is clear that this error diminished the longitudes of the Sun and Moon, employed by Ptolemy. The motion in longitude, which he attributed to the Stars, is too small by the arc described by the Sun in the time, equal to the error of Hipparchus in the length of the year.

In the time of Hipparchus, the tropical

year was 365.24234: this great astronomer supposed it 365,24667; the difference is 433″, and during this interval the Sun describes an arc of 47″; this, added to the annual precession of 111″, determined by Ptolemy, gives 158 for the precession, which he would have found, if he had computed from the length of the true tropical year, the error would then have been only 4″.

This remark has led to the examination of another question. It had been generally believed, that the catalogue of Ptolemy, was that of Hipparchus, reduced to his time by means of the annual precession of 111″. This opinion is founded on this circumstance, that the constant error in longitude of his Stars, disappear when reduced to the time of Hipparchus. But the explanation which we have given of the cause of this error, justifies Ptolemy from the reproach which has been imputed to him, of having taken the merit of Hipparchus to himself; and it seems right to believe him, when he asserts that he

has observed all the Stars of his own catalogue, even to the Stars of the sixth magnitude. He adds, at the same time, that he found very nearly the same positions of the Stars, relatively to the ecliptic, as Hipparchus, so that the difference between these two catalogues must have been very small. Thus, the observations of Ptolemy on the stars, and the true value which he has assigned to the evection, are proofs of his exactness as an observer. It is true, that the three equinoxes which he has observed, are inaccurate; but it appears that, too much prepossessed in favour of the exactness of the solar tables of Hipparchus, he made his observations of the equinoxes, at that time very difficult, coincide with them, as the derangement of his armillary might have been sufficient to explain the errors.

The astronomical edifice, raised by Ptolemy, subsisted near fourteen centuries, and now that it is entirely destroyed, his Almagest, considered as a depositary of ancient observations, is one of the most

precious monuments of antiquity. Ptolemy has not rendered less service to geography, in collecting all the known longitudes and latitudes of different places, and laying the foundation of the method of projections, for the construction of geographical charts. He composed a great treatise on optics, which has not been preserved, in which he explained the astronomical refractions: he likewise wrote treatises on the several sciences of chronology, music, gnomonics, and mechanics. So many labours, and on such a variety of subjects, manifest a very superior genius, and will ever obtain him a distinguished rank in the history of science. On the revival of astronomy, when his system gave way to that of nature, mankind avenged themselves on him for the despotism it had so long maintained; and they accused Ptolemy of having appropriated to himself the discoveries of his predecessors; but in his time, the works of Hipparchus, and of the astronomers of Alexandria, must have been sufficiently

known, to have rendered excusable, his not distinguishing what belonged to them from his own discoveries. As to the long continuation of his errors, it must be attributed to the same causes which replunged Europe into darkness. The fame of Ptolemy has met with the same fate as that of Aristotle and Descartes. Their errors were no sooner recognized, than a blind admiration gave way to an unjust contempt, for even in science itself, the most useful revolutions are not always exempt from passion and prejudice.

CHAP. III.

Of the Astronomy of the Arabs, Chinese and Persians.

The progress of astronomy in the school of Alexandria terminated with the labours of Ptolemy. This school continued to exist for five centuries, but the successors of Ptolemy and Hipparchus contented themselves with commenting on their works without adding to their discoveries. With the exception of two eclipses, recorded by Theon, and some observations of Theon of Athens, the phenomena of the heavens continued unobserved during a period of more than six hundred years. Rome, once the seat of valour, glory, and learning, did nothing useful to science.

The consideration that was always attached by the republic to eloquence and military talents, seduced the imagination to those pursuits: and science, offering no advantage, was necessarily neglected in the midst of conquests undertaken by ambition, and of internal commotions in which liberty expired and yielded to the despotism of the emperors. The division of the empire, the necessary consequence of its vast extent, brought on its fall, and the light of science, extinguished by the barbarians, was only again revived among the Arabians.

This people, exalted by fanaticism, after having extended its religion and its arms over a great part of the Earth, had no sooner reposed in peace, than it devoted itself to letters and science.

It, however, was but a short time before that they destroyed their most beautiful ornament, by burning the famous library of Alexandria.

In vain the philosopher Philoponus exerted himself for its preservation. If

these books, replied Omar, are conformable to the alcoran, they are useless; if they are contrary to it, they are detestable. Thus perished this immense treasure of erudition and genius. Repentance and regret soon followed this barbarous execution, for the Arabians were not long before they perceived their irreparable loss, and that they had deprived themselves of the most precious fruits of their conquests.

About the middle of the eighth century, the caliph Almansor gave great encouragement to astronomy, but among the Arabian princes who distinguished themselves for their love of the sciences, the most celebrated in history was Almamoun, of the family of the Abassides and son of the famous Aaron Rashid, so celebrated throughout Asia. Almamoun reigned in Bagdat in 814, having conquered the Greek emperor Michael III., he imposed on him, as one condition of peace, that he should have delivered to him the best books of Greece;—the Almagest was among the number, he caused it to be translated into

the Arabian language, and thus diffused the astronomical knowledge which had formerly caused the celebrity of the Alexandrine school. Not content with encouraging learned men by his liberality, he was himself an observer, and determined the obliquity of the ecliptic; he likewise caused a degree of the meridian to be measured on the vast plain of Mesopotamia.

The encouragement given to astronomy by this prince and his successors, produced a great number of astronomers, among whom Albategnius deserves to be placed the first. We are indebted to him for an observation of the obliquity of the ecliptic which corrected for refraction and parallax gives * 26°2182 for this obliquity of the ecliptic. All the Arabian observations give nearly the same result, from whence we deduce the secular variation about † 159.″

Albategnius found the annual motion of the equinoxes equal to ‡ 168.″3, and the length of the tropical year equal to 365.24056. The first of these elements is

* 23° 35′ 46″. † 51″.5. ‡ 51″.2.

too great by 14″, the second is too small by more than a minute and a half, but these errors depend entirely on the observations of Ptolemy which Albategnius compared with his own; he would have come nearer the truth had he used only those of Hipparchus.

This great astronomer improved greatly the theory of the Sun. He reduced the excentricity of the solar ellipse to 0.017325, the radius of the orbit being taken as unity. At the commencement of 1750 it was 0.016814. Its diminution in an interval of 870 years was therefore 0.00511. The theory of gravity, adopting the most probable value of the masses of the planets gives .003967. This difference is within the limits of the errors to which the observations of Albategnius were liable.

These same observations conducted him likewise to the discovery of the proper motion of the Sun's apogee, he observed its place to be * 24°.76 in *Gemini*, which was more advanced since the time of Hipparchus than it should have been from

the motion of the equinoxes only. According to our best tables the place of the apogee in 880 was * 26.°23, the observation of Albategnius was therefore only defective by † one degree and a half, which was a very precise determination for that age, considering the delicacy required to ascertain this element. These results are not only very valuable for their exactness, but particularly as they confirm the diminution of the excentricity of the solar orbit, demonstrated by the theory of gravitation and by the secular equation of the Moon; they likewise induce us to place great confidence in his determination of the obliquity of the ecliptic, which he relates to have been made with great care, by means of a radius of great length, and by taking all the precautions mentioned in the Almagest.

The most interesting part of the astronomy of the Arabs which have been preserved are these labours of Albategnius, in his work on the science of the stars, and two

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* 23°. † 36′.

eclipses of the Sun and one of the Moon observed by Ibn Junis, near Cairo, in 977, 978, and 979, which confirm the mean acceleration of the Moon. The Arabian astronomers chiefly occupied themselves in practical observations; they did not investigate the causes of the celestial phenomena, but retained without alteration the system of Ptolemy

The Persians, after having for a long time submitted to the same sovereigns as the Arabians, and professing the same religion, about the middle of the eleventh century, shook off the yoke of the Caliphs. About this time their calendar received a new form, by the care of the astronomer Omar Cheyam; it was founded on an ingenious intercalation, which consists in making in every thirty-three years six of them sextile.

Holagu Ilecoukan, one of their sovereigns, constructed a magnificent observatory, and entrusted the superintendance of it to Nassir Eddin. But no prince of this nation distinguished himself more for his

zeal than Uleg Beigh, whom we should place in the first rank of observers. He formed himself a catalogue of stars at Sarmacand, the capital of his dominions, and likewise the best tables of the Sun and planets which existed before the time of Tycho Brahe. He fixed the precession of the equinoxes at 159″, and determined the obliquity of the ecliptic with instruments of very elaborate construction, he found it equal to 261475.

A century and a half previous to this, the Chinese astronomy affords us several observations of the Sun, made with great care with a very high gnomon, by Cocheon King, a celebrated astronomer; from these Lacaille concluded the length of the year the same as we now adopt, and the obliquity of the ecliptic 26.1519, 1278, the epoch of these observations; from this results a secular variation of 153″. It is on these observations, and those of Albategnius, that I have founded my determination at 134.3.

The Chinese astronomy likewise men-

tions the occultation of several fixed stars by planets and a great many eclipses of the Sun and Moon. There no doubt exist in our libraries, manuscripts and other observations which would throw great light on the secular equations of the heavenly bodies, and on the masses of the planets, one of the principal things that remain unsettled in modern astronomy.

The investigation of these observations merit the attention of the learned in the oriental languages, for the great variations in the system of the world are not less interesting to be acquainted with, than the revolutions of empires.

CHAP. IV.

Of Astronomy in modern Europe.

It is to the Arabians that modern Europe is indebted for the first rays of light that dissipated the darkness in which it was enveloped during twelve centuries. They have transmitted to us the treasure of knowledge which they received from the Greeks who were themselves disciples of the Egyptians; but by a deplorable fatality the arts and sciences have disappeared among all these nations, as soon as they had communicated them.

Despotism has for a long period extended its barbarism over those beautiful countries where science first had its origin, and those names which formerly rendered them celebrated, are now unknown in them.

Alphonso, king of Castille, was one of the first sovereigns who encouraged the revival of astronomy in Europe. This science can reckon but few such zealous protectors; but he was ill seconded by the astronomers whom he had assembled at a considerable expence, and the tables which they published did not answer to the great cost they had occasioned.

Endowed with a correct judgment, Alphonso was shocked at the confusion of the circles, in which the celestial bodies were supposed to move; he felt that the expedients employed by nature ought to be more simple. "If the Deity," said he, "had asked my advice, these things would have been better arranged." By these words, which are taxed with impiety, he meant to express that mankind were still far from knowing the true mechanism of the universe.

In the time of Alphonso, Europe was indebted to the encouragement of Frederic II. Emperor of Germany, for the first Latin translation of the Almagest of Pto-

lemy, which was made from the Arabic version.

We are now arrived at that celebrated epoch when astronomy, escaping from the narrow sphere which had hitherto confined it, raised itself by a rapid and continued progress to the height where we now behold it. Purbech, Regiomontanus, and Waltherus prepared the way to these prosperous days of the science, and Copernicus gave them birth by the fortunate explanation of the celestial phenomena, by means of the motion of the Earth on its axis, and round the Sun.

Shocked, like Alphonso, at the extreme complication of the system of Ptolemy, he tried to find among the ancient philosophers a more simple arrangement of the universe. He found that many of them had supposed Venus and Mercury to move round the Sun: that Nicetas, according to Cicero, made the Earth revolve on its axis, and by this means freed the celestial sphere from that inconceivable velocity which must be attributed to it to accomplish its

diurnal revolution. He learnt from Aristotle and Plutarch that the Pythagoricians had made the Earth and planets move round the Sun, which they placed in the centre of the universe. These luminous ideas struck him; he applied them to the astronomical observations which time had multiplied, and had the satisfaction to see them yield, without difficulty, to the theory of the motion of the Earth. The diurnal revolution of the heavens was only an illusion due to the rotation of the Earth, and the precession of the equinoxes, is reduced to a slight motion of the terrestrial axis. The circles, imagined by Ptolemy, to explain the alternate direct and retrograde motions of the planets, disappeared. Copernicus only saw in these singular phenomena, the appearances produced by the motion of the Earth round the Sun, with that of the planets: and he concluded, from hence, the respective dimensions of their orbits, which, till then, were unknown. Finally, every thing in this system announced that

beautiful simplicity in the expedients of nature, which delights so much when we are fortunate enough to discover it. Copernicus published it in his work, *On the Celestial Revolutions*; not to shock received prejudices, he presented it under the form of an hypothesis. "Astronomers," said he, "in his dedication to Paul III., being permitted to imagine circles, to explain the motion of the stars, I thought myself equally entitled to examine if the supposition of the motion of the Earth, would render the theory of these appearances more exact and simple."

This great man did not witness the success of his work. He died suddenly by the rupture of a blood vessel, at the age of seventy-one years, a few days after receiving the first proof. He was born at Thorn, in Polish Prussia, the 19th of February, 1473. After learning the Greek and Latin languages, he went to continue his studies at Cracovia. Afterwards, induced by his taste for astronomy, and by the reputation which Regiomontanus had acquired,

he undertook a journey to Italy, where this science was taught with success: being greatly desirous to render himself illustrious by the same career, he followed the lessons of Dominic Maria, at Bologna. When arrived at Rome, his talents obtained him the place of professor: he afterwards quitted this city, to establish himself at Fravenberg, where his uncle, then Bishop of Warmia, made him a canon. It was in this tranquil abode, that by thirty-six years of observation and meditation, he established his theory of the motion of the Earth. At his death, he was buried in the cathedral of Fravenberg, without any pomp or epitaph; but his memory will exist as long as the great truths which he has again introduced with such evidence, as to have at length dissipated the illusions of the senses, and surmounted the difficulties which ignorance of the laws of mechanics had opposed to them.

These truths had yet to vanquish obstacles of another kind, and which, arising from a respected source, would have sti-

fled them if the rapid progress of all the mathematical sciences had not concurred to support them.

Religion was invoked to destroy an astronomical system, and one of its defenders, whose discoveries did honor to his age and country, was tormented by repeated prosecutions. Bethicus, the disciple of Copernicus, was the first who adopted his ideas; but they were not in great estimation till towards the beginning of the seventeenth century, and then they owed it principally to the labours and misfortunes of Galileo.

A fortunate accident had made known the most wonderful instrument ever discovered by human ingenuity, and which, by giving to astronomical observations a precision and extent hitherto unhoped for, displayed in the heavens new inequalities, and new worlds. Galileo hardly knew of the first trials of the telescope, before he bent his mind to bring it to perfection. Directing it towards the Stars, he discovered the four satellites of Jupiter, which

shewed a new analogy between the Earth and planets; he afterwards observed the phases of Venus, and from that moment he no longer doubted of its motion round the Sun. The milky way displayed to him an infinite number of small stars, which the irradiation confounds to the naked eye, in a white and continued light; the luminous points which he perceived beyond the line which separated the light part of the Moon from the dark, made him acquainted with the existence and height of its mountains. At length he observed the appearances occasioned by Saturn's ring, the spots and rotation of the Sun. In publishing these discoveries, he shewed that they proved incontestibly, the motion of the Earth; but the idea of this motion was declared heretical by a congregation of cardinals; and Galileo, its most celebrated defender, was cited to the tribunal of the inquisition, and compelled to retract this theory, to escape a rigorous prison.

One of the strongest passions is the love

of truth, in a man of genius. Full of the enthusiasm which a great discovery inspires, he burns with ardour to disseminate it, and the obstacles which ignorance and superstition, armed with power, oppose to it, only irritate and increase his energy. Galileo, convinced by his own observations of the motion of the Earth, had long meditated a new work, in which he proposed to develope the proofs of it. But to shelter himself from the persecution of which he had escaped being the victim, he proposed to present them, under the form of dialogues between three interlocutors, one of whom defended the system of Copernicus, combated by a Peripatetician. It is obvious, that the advantage would rest with the defender of this system; but, as Galileo did not decide between them, and gave as much weight as possible to the objections of the partisans of Ptolemy, he had a right to expect that tranquillity which his age and labours merited.

The success of these dialogues, and the triumphant manner with which all the

difficulties against the motion of the Earth were resolved, roused the inquisition. Galileo, at the age of seventy, was again cited before this tribunal. The protection of the grand Duke of Tuscany, could not prevent his appearance. He was confined in a prison, where they required of him a second disavowal of his sentiments, threatening him with the punishment incurred by contumacy, if he continued to teach the system of Copernicus.

He was compelled to sign this formula of abjuration:

"*I Galileo, in the seventieth year of my*
"*age, brought personally to justice, being*
"*on my knees, and having before my eyes*
"*the holy evangelists, which I touch with*
"*my own hands, with a sincere heart and*
"*faith; I abjure, curse, and detest, the*
"*absurdity, error, and heresy, of the mo-*
"*tion of the Earth,*" &c.

What a spectacle! A venerable old man, rendered illustrious by a long life, consecrated to the study of nature, abjuring on his knees, against the testimony of

his own conscience, the truth which he had so evidently proved. A decree of the inquisition, condemned him to a perpetual prison. He was released after a year, at the solicitations of the grand duke; but, to prevent his withdrawing himself from the power of the inquisition, he was forbidden to leave the territory of Florence.

Born at Pisa, in 1564, he gave early indications of those talents which were afterwards developed. Mechanics owe to him many discoveries, of which the most important is the theory of falling bodies.

Galileo was occupied with the libration of the Moon, when he lost his sight; he died three years afterwards, at Arcetre, in 1642, regretted by all Europe, which he left enlightened by his labours, and indignant at the judgment passed against so great a man, by an odious tribunal.

While this passed in Italy, Kepler, in Germany, developed the laws of the planetary motions. But, previous to the account of his discoveries, it is necessary to

look back and to describe the progress of astronomy in the north of Europe, after the death of Copernicus.

The history of this science presents at this epoch, a great number of excellent observers. One of the most illustrious, was William IV. Landgrave of Hesse-Cassel. He had an observatory built at Cassel, which he furnished with instruments, constructed with care, and with which he observed a long time. He procured two celebrated astronomers, Rothman and Juste Byrye; and Tycho owed to his pressing solicitations, the advantages which Frederic, King of Denmark, obtained for him.

Tycho Brahe, who was one of the greatest observers that ever existed, was born at Knucksturp, in Norway. His taste for astronomy was manifested at the age of fourteen years, on the occasion of an eclipse of the Sun, which happened in 1560. At this age, when reflection is so rare, the justice of the calculation which announced this phenomenon, inspired him

with an anxious desire to know its principles; and this desire was still further increased by the opposition of his preceptor and family. He travelled to Germany, where he formed connections of correspondence and friendship, with the most distinguished persons, who pursued astronomy either as a profession, or amusement, and particularly with the Landgrave of Hesse-Cassel, who received him in the most flattering manner.

On his return to his own country, he was fixed there by his sovereign, Frederic, who gave him the little island of Huena at the entrance of the Baltic. Tycho built a celebrated observatory there, which was called Uranibourg. There, during an abode of twenty-one years, he made a prodigious mass of observations, and many important discoveries. At the death of Frederic, envy, then unrestrained, compelled Tycho to leave his retreat. His return to Copenhagen did not appease the rage of his prosecutors; the minister, Walchendorp, (whose name, like that of all men

who have abused the power entrusted to them, ought to be handed down to the execration of all ages,) forbad him to continue his observations. Fortunately, Tycho found a powerful protector in the Emperor Rodolph II. who settled on him a considerable pension, and lodged him commodiously at Prague. He died suddenly at this city, on the 24th of October, 1601, in the midst of his labours, and at an age when astronomy might have expected great services from him.

The invention of new instruments, and new improvements, added to the old ones a much greater precision in observation; a catalogue of stars very superior to those of Hipparchus, and Ulugh Beigh; the discovery of that inequality of the Moon, which is called variation; that of the inequalities of the motion of the nodes, and of the inclination of the lunar orbit; the interesting remark, that comets are beyond this orbit; a more perfect knowledge of astronomical refractions; finally, very numerous observations of the planets, which

have served as the basis of the discoveries of Kepler. Such are the principal services which Tycho Brahe has rendered astronomy. Struck with the objections which the adversaries of Copernicus made to the motion of the Earth, and perhaps influenced by the vanity of wishing to give his name to an astronomical system, he mistook that of nature. According to him, the Earth is immovable in the centre of the universe; all the Stars move every day round the axis of the world; and the Sun, in its annual revolution, carries with it the planets. In this system, already known, the appearances are the same as in that of the motion of the Earth. We may, in general, consider any point we chuse; for example, the centre of the Moon as immovable, provided we assign the motion with which it is animated, in a contrary direction to all the stars.

But, is it not physically absurd to suppose the Earth immovable in space, while the Sun carries with it the planets in which it is included? How could the distance

from the Earth to the Sun, which agrees so well with the duration of its revolution in the hypothesis of the motion of the Earth, leave any doubt of the truth of this hypothesis, in a mind constituted to feel the force of analogy. It must be confessed, that Tycho, though a great observer, was not fortunate in his research after causes; his unphilosophical mind had even imbibed the prejudices of astrology, which he tried to defend.

It would be, however, unjust to judge him with the same rigor as one who should refuse at present to believe the motion of the Earth, confirmed by the numerous discoveries made in astronomy since that period.

The difficulties which the illusions of the senses opposed to this theory, were not then completely removed. The apparent diameter of the fixed stars, greater than their annual parallax, gave to these stars in this theory, a real diameter, greater than that of the terrestrial orbit. The telescope, by reducing them to luminous

points, made this improbable magnitude disappear. It could not be conceived how these bodies, detached from the Earth, could follow its motion. The laws of mechanics have explained these appearances; they have proved, what Tycho had again made doubtful, that a body, falling from a considerable height, and abandoned to the action of gravity alone, ought to fall very nearly in a vertical line, only deviating to the east, by a quantity difficult to estimate accurately by observation from its minuteness, so that at present there is as much difficulty in proving the motion of the Earth by a direct experiment, as formerly existed to prove that it should be insensible.

In his later years, Tycho Brahe had Kepler for a disciple and assistant. He was born in 1571, at Viel, in the duchy of Wirtemberg, and was one of those extraordinary men whom nature grants now and then to the sciences, to bring to light those great theories which have been prepared by the labour of many centuries.

The career of the sciences did not appear to him proper to satisfy the ambition he felt of rendering himself illustrious; but the ascendancy of his genius, and the exhortations of Maestlinus, led him to astronomy: and he entered into the pursuit with all the activity of a mind passionately desirous of glory.

The philosopher, endowed with a lively imagination, and impatient to know the causes of the phenomena which he sees, often obtains a glimpse of it, before observation can conduct him to it. Doubtless he might, with greater certainty, ascertain the cause from the phenomena; but the history of science proves to us, that this slow progress has not always been that of inventors.

What rocks has he to fear, who takes his imagination for his guide!

Prepossessed with the cause which it presents to him, instead of rejecting it when contradicted by facts, he alters them to make them agree with his hypotheses; he mutilates, if I may be allowed the expression, the work of nature

to make it resemble that of his imagination, without reflecting that time destroys with one hand these vain phantoms, and with the other confirms the results of calculation and experience.

The philosopher who is really useful to the cause of science, is he, who, uniting to a fertile imagination, a rigid severity in investigation and observation, is at once tormented by the desire of ascertaining the cause of the phenomena, and by the fear of deceiving himself in that which he assigns.

Kepler owed the first of these advantages to nature, and the second to Tycho Brahe. This great observer, whom he went to see at Prague, and who had discovered the genius of Kepler, in his earliest works, notwithstanding the mysterious analogies of numbers and figures with which it was filled, exhorted him to devote his time to observation, and procured him the title of imperial mathematician.

The death of Tycho, which happened a few years afterwards, put Kepler in possession of his valuable collection of obser-

vations, of which he made a most noble use, founding on them three of the most important discoveries that have been made in natural philosophy.

It was an opposition of Mars which determined Kepler to employ himself, in preference, on the motions of this planet. His choice was fortunate in this circumstance, that the orbit of Mars, being one of the most excentric of the planetary system, the inequalities of his motion were more perceptible, and therefore led to the discovery of their laws with greater facility and precision. Though the theory of the motion of the Earth had made the greater part of those circles with which Ptolemy had embarrassed astronomy, disappear, yet Copernicus had substituted many others to explain the real inequalities of the celestial bodies.

Kepler, deceived like him, by the opinion that their motions ought to be circular and uniform, tried a long time to represent those of Mars, in this hypothesis. Finally, after a great number of trials,

which he has related in detail in his famous work called *Stella Martis*, he overcame the obstacle, which an error, supported by the suffrage of every period, had opposed to him; he discovered that the orbit of Mars is an ellipse, of which the Sun occupies one of the foci, and that the motion of the planet is such, that the radius vector, drawn from its centre to that of the Sun, describes equal areas in equal times. Kepler extended these results to all the planets, and published from this theory, in 1626, the Rudolphine tables, for ever memorable in astronomy, as being the first founded on the true laws of the planetary motions.

Without the speculations of the Greeks, on the curves formed from the section of a cone by a plane, these beautiful laws might have been still unknown. The ellipse being one of these curves, its oblong figure gave rise, in the mind of Kepler, to the idea of supposing the planet Mars, whose orbit he had discovered to be oval, to move on it, and soon, by means of the

numerous properties which the ancient geometricians had found in the conic sections, he became convinced of the truth of this hypothesis. The history of the sciences offers us many examples of these applications of pure geometry, and of its advantages; for every thing is connected in the immense chain of truths, and often a single observation has been sufficient to shew the connection between a proposition apparently the most sterile, and the phenomena of nature which are only mathematical results of general laws.

The perception of this truth probably gave birth to the mysterious analogies of the Pythagoricians: they had seduced Kepler, and he owed to them one of his most beautiful discoveries. Persuaded that the mean distances of the planets from the Sun, ought to be regulated conformably to these analogies, he compared them a long time, both with the regular geometrical solids, and with the intervals of tones. At length, after seventeen years of meditations and calculation, conceiving

the idea of comparing the powers of the numbers which expressed them, he found that the squares of the times of the planetary revolutions, are to each other as the cubes of the major axes of their orbits; a most important law, and which he had the advantage of observing in the system of satellites of Jupiter, and which extends to all the systems of satellites.

We might be astonished that Kepler should not have applied the general laws of elliptic motion to comets. But, misled by an ardent imagination, he lost the clue of the analogy, which should have conducted him to this great discovery. The comets, according to him, being only meteors, engendered in ether; he neglected to study their motions, and thus stopped in the middle of the career which was open to him, abandoning to his successors a part of the glory which he might yet have acquired. In his time, the world had just begun to get a glimpse of the proper method of proceeding in the search of truth, at which genius only arrived by

its discoveries. Instead of passing slowly y a succession of inductions, from insulated phenomena, to others more extended, and from these to the general laws of nature ; it was more easy and more agreeable to subject all the phenomena to the relations of convenience and harmony, which the imagination could create and modify at pleasure.

Thus, Kepler explained the disposition of the solar system, by the laws of musical harmony. We behold him even in his latest works, amusing himself with these chimerical speculations, even so far as to regard them as the "*life and soul*" of astronomy. He has deduced from them the excentricity of the terrestrial orbit, the density of the Sun, its parallax, and other results ; the inaccuracy of which, now discovered, is a proof of the errors to which we expose ourselves, in deviating from the rout traced by observation.

After having destroyed the epicycles which Copernicus had preserved ; after having determined the curve which the planets

describe round the Sun, and discovered the laws of their motion; Kepler too near to the principle from which these laws were derived, not to foresee it. Attempts to discover this principle, often exercised his active imagination; but the moment was not yet arrived, to make this last step, a more profound knowledge of mechanics, and a more perfect state of geometry.

However, amidst the fruitless trials of Kepler, and his numerous errors, the connection of facts conducted him to correct opinions on this subject, in the work in which he published his principal discoveries.

"Gravity," he says, "in his *Commentary on Mars*, is only a mutual and corporeal affection between similar bodies. "Heavy bodies do not tend to the centre "of the world, but to that of the round "body, of which they form a part; and "if the Earth were not spherical, heavy "bodies would not fall towards its centre, "but towards different points."

If the Moon and Earth were not re-

tained at their respective distances, they would fall upon each other, the Moon passing through to $\frac{53}{54}$ of the distance, and the Earth passing through the remainder, supposing them equally dense. He believed also, that the attraction of the Moon was the cause of the tides, and he suspected, that the irregularities of the lunar motion were produced by the combined actions of the Sun and Earth on the Moon.

Astronomy likewise owes to Kepler, many useful discoveries. His work on optics, is full of new and interesting matter; he there explains the mechanism of vision, which was unknown before him. He assigned the true cause of the *lumiere cendrée* of the Moon; but he gave the honor of this discovery to his master, Maestlinus, entitled to notice from this discovery, and from having recalled Kepler to astronomy, and converted Galileo to the system of Copernicus.

Finally, Kepler, in his work entitled, *Stereometria Daliorum*, has presented some conceptions on affinity which have

influenced the revolution experienced by geometry towards the end of the last century.

With so many claims to admiration, this great man lived in misery, while judicial astrology, every where honored, was magnificently recompensed. The astronomers of his time, Descartes himself and Galileo, who might have received the greatest advantage from his discoveries, do not appear to have perceived their importance.

Fortunately the enjoyment which a man of genius receives from the truths which he discovers, and the prospect of a just and grateful posterity, console him for the ingratitude of his contemporaries.

Kepler had obtained pensions which were always ill paid : going to the diet of Ratisbon to solicit his arrears, he died in that city the 15th of November 1630. He had in his latter years the advantage of seeing the discovery of logarithms, and making use of them. This was due to Nepier, a Scottish baron ; it is an admirable contriv-

ance, an improvement on the ingenious algorithm of the Indians, and which, by reducing to a few days the labour of many months, we may almost say doubles the life of astronomers, and spares them the errors and disgusts inseparable from long calculations;—an invention so much the more gratifying to the human mind, as it is entirely due to its own powers: in the arts man makes use of the materials and forces of nature to increase his powers, but here all is his own work.

The labours of Huygens followed soon after those of Kepler and Galileo. Very few men have deserved so well of the sciences, by the importance and sublimity of their researches. The application of the pendulum to clocks is one of the most beautiful acquisitions which astronomy and geography have made, and to which fortunate invention, and to that of the telescope, the theory and practice of which Huygens considerably improved, they owe their rapid progress.

He discovered, by means of excellent

object-glasses which he succeeded in constructing, that the singular appearances of Saturn were produced by a very thin ring, with which the planet is surrounded: his assiduity in observing made him discover one of the satellites of Saturn.

He made numerous discoveries in geometry and mechanics; and if this extraordinary genius had conceived the idea of combining his theorems on centrifugal forces with his beautiful investigation on involutes, and with the laws of Kepler, he would have preceded Newton in his theory of curvilinear motion, and in that of universal gravitation. But it is not in such approximations that discovery consists.

Towards the same time, Hevelius rendered himself useful to astronomy, by his immense labours. Few such indefatigable observers have existed; it is to be regretted that he would not adopt the application of telescopes to quadrants, an invention which gave a precision previously unknown to astronomy.

At this epoch astronomy received a new impulse from the establishment of learned societies.

Nature is so various in her productions and phenomena, of which it is so difficult to ascertain the causes, that it is requisite for a great number of men to unite their intellect and exertions to comprehend and develope her laws. This union is particularly requisite when the sciences in extending approximate, and require mutual support from each other.

It is then, that the natural philosopher has recourse to geometry, to arrive at the general causes of the phenomena which he observes, and the geometrician in his turn interrogates the philosopher, in order to render his own investigation useful, by applying them to experience: and to open in these applications a new road in analysis. But the principal advantage of learned societies is the philosophical feeling on every subject which is introduced into them, and from thence diffuses itself over the whole nation. The insulated philoso-

pher may resign himself without fear to the spirit of system; he only hears contradiction at a distance; but in a learned society the shock of systematic opinions at length destroys them, and the desire of mutually convincing each other establishes between the members an agreement only to admit the results of observation and calculation. Thus experience has proved that since the origin of these establishments true philosophy has been generally extended.

By setting the example of submitting every opinion to the test of severe scrutiny, they have destroyed prejudices which had so long reigned among the sciences, and in which the highest intellects of the preceding ages had participated. Their useful influence on opinion accumulated in our own time, with an enthusiasm which at other periods would have perpetuated them. Finally, it is among them or by the encouragement they offer that those grand theories have been formed which are placed above the reach of the vulgar

by their comprehensiveness; and which, extending themselves by their numerous occasions in which they are applicable, to nature and to the arts, are inexhaustible sources of delight and intelligence.

Of all the learned societies, the two most celebrated for the number and importance of their discoveries in the sciences, and particularly in astronomy, are the Academy of Sciences in Paris, and the Royal Society in London.

The first was created in 1666, by Louis XIV. who foresaw the lustre which the arts and sciences were to diffuse over his reign. This monarch, worthily seconded by Colbert, invited many learned strangers to fix themselves in his capital. Huygens availed himself of this flattering invitation; he published his admirable work *De horologio oscillatorio*, in the midst of the academy, of which he was one of the first members. He would have finished his days in this country, had it not been for the disastrous edict which, towards the end of the last century, deprived

France of so many valuable citizens. Huygens, departing from a country in which the religion of his ancestors was proscribed, retired to the Hague, where he was born the 14th of April, 1629, and died there the 15th of June, 1695.

Dominic Cassini was likewise induced to go to Paris, by the benefits of Louis XIV. During forty years of useful labours, he enriched astronomy with a crowd of discoveries: such are the theory of the satellites of Jupiter, the motions of which he determined from observations of their eclipses; the discovery of the four satellites of Saturn; those of the rotation of Jupiter, of the belts parallel to his equator, of the rotation of Mars, of the zodiacal light, a very approximate knowledge of the Sun's parallax, a very exact table of refractions, and, above all, the complete theory of the libration of the Moon.

The great number of astronomical academicians of extraordinary merit, and the limits of this historical abridgment, do not permit me to give an account of their

labors; I shall content myself with observing that the application of the telescope to the quadrant, the invention of the micrometer and heliometer, the successive propagation of light, the magnitude of the Earth, its ellipticity, and the diminution of gravity at the equator, are all discoveries due to the Academy of Sciences.

Astronomy does not owe less to the Royal Society of London, the origin of which is a few years anterior to that of the Academy of Sciences. Among the astronomers which it has produced, I shall cite Flamstead, one of the greatest observers that have ever appeared. Halley, rendered illustrious by his travels, undertaken for the advantage of science, by his beautiful investigation concerning comets, which enabled him to discover the return of the comet in 1759; and by the ingenious idea of employing the transit of Venus over the Sun, in the determination of its parallax. I shall mention, lastly, Bradley, the model for observers, and who will be for ever celebrated for two

of the most beautiful discoveries ever made in astronomy, the aberration of the fixed stars, and the nutation of the axis of the Earth.

When the application of the pendulum to clocks, and of telescopes to quadrants, had rendered the slightest changes in the position of the celestial bodies perceptible to observers, they endeavoured to determine the annual parallax of the fixed stars; for it was natural to suppose, that *so great an extent* as the diameter of the terrestrial orbit, would be sensible even at the distance of these stars. Observing them carefully, at every season of the year, there appeared slight variations; sometimes favorable, but more frequently contrary to the effects of parallax. To determine the law of these variations, an instrument of great radius, and divided with extreme precision, was requisite. The artist who executed it, deserves to partake of the glory of the astronomer who owed his discovery to him. Graham, a famous English watch-maker, constructed a great

sector, with which Bradley discovered the aberration of the fixed stars, in the year 1727. To explain it, this great astronomer conceived the fortunate idea of combining the motion of the Earth with that of light, which Roemer had discovered at the end of the last century, by means of the eclipses of Jupiter's satellites. We should be surprised that none of the distinguished philosophers who then existed, and who knew the motion of light, should have paid any attention to the very simple effects which result from it, in the apparent position of the fixed stars. But, the human mind, so active in the formation of systems, has almost always waited till observation and experience have acquainted it with important truths, which its powers of reasoning alone might have discovered.

It is thus that the invention of telescopes has followed by more than three centuries that of lenses, and even then was only due to accident.

In 1745, Bradley discovered by obser-

vation, the nutation of the terrestrial axis. In all the apparent variations of the fixed stars, observed with extraordinary care, he perceived nothing which indicated a perceptible parallax. The measures of the degrees of the terrestrial meridian, and of the pendulum, multiplied in different parts of the globe, of which France gave the example, by measuring the whole arc of the meridian, which crosses it, and by sending the academician to the north and to the equator, to observe the magnitude of these degrees, and the intensity of the force of gravity. The arc of the meridian, comprised between Dunkirk and Barcelona, determined by very precise observations, and forming the base of the most natural and simple system of measures; the voyages undertaken to observe the two transits of Venus over the Sun's disk, in 1761 and 1769, and the exact knowledge of the dimensions of the solar system, which has been derived from these voyages; the invention of achromatic telescopes, of chronometers, of the sextant

and repeating circle, the discovery of the planet Uranus, by Herschel, in 1781; that of its satellites, and of two new satellites of Saturn, due to the same observer, all the astronomical theories being brought to perfection, and all the celestial phenomena, without exception, being referred to the principle of universal gravitation. These, with the discoveries of Bradley, are the principal obligations which astronomy owes to our century, which, with the preceding, will always be considered as the most glorious epoch of the science.

CHAP. V.

Of the Discovery of universal Gravitation.

After having shewn by what successive efforts the human mind has attained the knowledge of the celestial motions, it only remains to consider the means by which it has arrived at the general principle, on which these laws depend. Descartes was the first who endeavoured to reduce the motions of the heavenly bodies to some mechanical principle. He imagined vortices of subtle matter, in the centre of which he placed these bodies. The vortex of the Sun forced the planet into motion; that of the planet, in the same manner, forced its satellite to revolve round it. The motion of comets traversing the heavens in all directions, destroyed these vortices, as they had before destroyed the

solid christalline spheres of the ancient astronomers. Thus, Descartes was no happier in his mechanical, than Ptolemy in his astronomical theory. But their labours have not been useless to science. Ptolemy has transmitted to us, through fourteen centuries of ignorance, the few astronomical truths which the ancients had discovered. Descartes, born at a later period, and at a time when the universal curiosity was excited, which he himself had increased, by substituting in the place of ancient errors, others more seducing, and resting on the authority of his geometrical discoveries, was enabled to destroy the empire of Aristotle and Ptolemy, which might have stood the attack of a more careful philosopher; but by establishing as a principle, that we should begin by doubting of every thing, he himself warned us to examine his own system with severity, which could not long resist the new truths that were opposed to it. It was reserved for Newton to teach us the general principles of the

heavenly motions. Nature not only endowed him with a profound genius, but placed his existence in a most fortunate period. Descartes had changed the face of the mathematical sciences, by the application of algebra to the theory of curves and variable functions. The geometry of infinites, of which this theory contained the germ, began to appear in various places. Wallis, Wren, and Huygens, had discovered the laws of motion; the discoveries of Galileo, on falling bodies, and of Huygens on evolutes and centrifugal force, led to the theory of motion in curves; Kepler had determined those described by the planets, and had formed a remote conception of universal gravitation; and finally, Hook had distinctly perceived that their motion was the result of a projectile force, combined with the attractive force of the Sun. The science of celestial mechanics wanted nothing more to bring it to light, but the genius of man, who, by generalizing these discoveries, should be capable of perceiving the

law of gravitation: it is this which Newton accomplished in his immortal work on the mathematical principles of natural philosophy. This philosopher, so deservedly celebrated, was born at Woolstrop, in England, in the latter end of the year 1642, the year in which Galileo died. His first success in his early studies, announced his future reputation; a cursory perusal of elementary books, was sufficient for him to comprehend them; he next read the geometry of Descartes, the optics of Kepler, and the arithmetic of infinites, by Wallis, but soon aspiring to new inventions, he was, before the age of twenty-seven, in possession of his method of fluxions, and his theory of light. Anxious for repose, and averse to literary controversy, he delayed publishing his discoveries. His friend and preceptor, Dr. Barrow, exerted himself in his favor, and obtained for him the situation of professor of mathematics in the university of Cambridge; it was during this period, that, yielding to the request of Halley, and the solicitations of

the Royal Society, he published his *Principia*. The university, of which he was a member, chose him for their representative, in the conventional parliament of 1788, and for that which was convened in 1701. *He was knighted and appointed* director of the mint, by Queen Anne: he was elected president of the Royal Society in 1703, which dignity he enjoyed till his death, in 1727. During the whole of his life he obtained the most distinguished consideration, and the nation to whose glory he had so much contributed, decreed him at his death public funeral honours.

In 1666, Newton retired into the country, and, for the first time, directed his thoughts to the system of the world. The descent of heavy bodies, which appears nearly the same at the summit of the highest mountains as at the surface of the Earth, suggested to him the idea, that gravity might extend to the Moon, and that being combined with some motion of projection, it might cause it to describe its elliptic orbit round the Earth. To verify

this conjecture, it was necessary to know the law of the diminution of gravity. Newton considered, that if the Moon was retained in its orbit by the gravity of the Earth, the planets should also be retained in their orbits by their gravity towards the Sun, and demonstrated this by the law of the areas being proportional to the times. Now it results from the relation of the squares of the times to the cubes of the greater axis of their orbits, that their centrifugal force, and consequently their tendency to the Sun, diminishes inversely as the squares of the distances from this body. Newton, therefore, transferred to the Earth this law of the diminution of the force of gravity, and reasoning from the experiments of falling bodies, he determined the height which the Moon, abandoned to itself, would fall in a short interval of time. This height is the versed sine of the arc which it describes in the same interval; and this quantity the lunar parallax gives in parts of the radius of the Earth, so that, to com-

pare the law of gravitation with observation, it was necessary to know the magnitude of this radius; but Newton having, at that time, an erroneous estimate of the terrestrial meridian, obtained a different result from what he expected, and suspecting that some unknown forces united themselves with the gravity of the Moon, he abandoned his original idea. Some years afterwards, a letter from Dr. Hook induced him to investigate the nature of the curve described by projectiles round the centre of the Earth. Picard had lately finished the measure of a degree in France, and Newton found, by this measure, that the Moon was retained in its orbit by the force of gravity alone, supposed to vary inversely as the square of the distance. By this law he found that bodies in their fall, describe ellipses, of which the centre of the Earth occupies one of their foci, and then, considering that the planetary orbits are likewise ellipses, having the Sun in one of their foci, he had the satisfaction to see, that the

solution which he had undertaken from curiosity, could be applied to the greatest objects in nature. He arranged the several propositions relative to the elliptic motions of planets, and Dr. Halley having induced him to publish them, he composed his grand work, the Principiæ, which appeared in 1687. These details, which have been transmitted to us by his friend and cotemporary Dr. Pemberton, prove that this great philosopher had, so early as 1666, discovered the principal theorems on centrifugal force, which Huygens published six years afterwards, at the end of his work De Horologio Oscillatorio ; for, indeed it is highly probable that the author of the method of fluxions, who seems then to have been in possession of it, should easily have discovered these theorems. Newton arrived at the law of the diminution of gravity, by the relation which subsists between the squares of the periodic times, and the cubes of the greater axes of their orbits, supposed circular. He demonstrated that this relation exists

in elliptic orbits generally, and that it indicates an equal gravity of the planets towards the Sun, supposing them at an equal distance from its centre. The same equality of gravity towards the principal planet, exists likewise in all the systems of satellites, and Newton verified it on terrestrial bodies, by very accurate experiments.

This great geometrician, by considering this question generally, demonstrated that a projectile can move in any conic-section whatever, in consequence of a force directed towards its centre, and varying reciprocally as the square of the distances. He investigated the different properties of motion in this species of curves; he determined the conditions requisite for the section to be a circle, an ellipse, a parabola, or an hyperbola, which conditions depend entirely on the velocity and primitive position of the body.

Any velocity, position, and initial direction of a body being given, Newton assigned the conic section which the body should de-
[illegible], and in which it ought consequently to

move, which answers the reproach which John Bernouilli made him of not having demonstrated, that the conic sections are the only curves which a body, solicited by a force varying reciprocally as the squares of the distance, can describe. These investigations, applied to the motion of comets, informed him that these bodies move round the Sun, according to the same laws as the planets, with the difference only of their ellipses being very excentric; and he gave the means of determining by observation, the elements of these ellipses.

He learned from the comparison of the distance and duration of the revolutions of satellites, with those of the planets, the respective densities and masses of the Sun, and of planets accompanied by satellites, and the intensity of the force of gravity at their surface.

By considering that the satellites move round their planets very nearly, as if the planets were immovable, he discovered that all these bodies obey the same force of gravity towards the Sun.

The equality of action and reaction, did not permit him to doubt, but that the Sun gravitated towards the planets, and these towards their satellites; and even that the Earth is attracted by all the bodies that rest upon it. He extended this proposition afterwards by analogy, to all the celestial bodies, and established as a principle, *that all particles of matter attract each other directly as their mass, and inversely as the square of their distance.*

Arrived at this principle, Newton saw that the great phenomena of the system of the world might be deduced from it. By considering gravity at the surfaces of the celestial bodies, as the result of the attractions of all their particles, he ascertained these remarkable truths, that the attracting force of a body, or of a spherical stratum, on a point placed without it, is the same as if its mass was compressed into its centre; and that a point placed within a spherical stratum, or generally any stratum terminated by two elliptic sur-

faces, similar and similarly situated, is equally attracted on every side.

He proved that the motion of rotation of the Earth, ought to have flattened it in the direction of the poles, and he determined the law of the variation of the degrees and of gravity, supposing it homogeneous.

He saw that the action of the Sun and Moon on the terrestrial spheroid ought to produce a motion in its axis of rotation, to make the equinoxes retrograde, to elevate the waters of the ocean, and to produce in this great fluid mass the oscillations which are observed under the name of tides.

Lastly, he was convinced that the lunar irregularities were produced by the combined action of the Sun and Earth on this satellite. But with the exception of what concerns the elliptic motion of the planets and comets, the attraction of spherical bodies, and the intensity of gravity at the surface of the Sun, and of those planets that are accompanied by satellites, all

these discoveries were only sketched by Newton. His theory of the form of the planets is limited by the supposition of their homogenity: his solution of the problem of the precession of the equinoxes, though very ingenious, is, notwithstanding the apparent agreement of his result with observation, in many respects defective; in the great number of the perturbations of the celestial motions, he has only considered those of the lunar motion, of which the most considerable, the evection, has escaped his investigation. He has perfectly established the existence of the principle which he discovered, but the developement of its consequences and its advantages, has been the work of the successors of this great geometrician. The state of imperfection in which the infinitesmal calculus must have been in the hands of its inventor, has not permitted him to resolve completely the difficult problems which the theory of the system of the world presents; and he has been often obliged to give conjectures, at least uncertain till they have

been verified by a rigorous calculation. Notwithstanding these inevitable defects, the importance and extent of his discoveries, the great number of original and profound conceptions, which have been the germ of the most brilliant theories of the geometricians of this century, and arranged with much elegance, insures to his *Principia* a pre-eminence over all other productions of human intellect.

The case is not the same with the sciences as with literature: this has limits which a man of genius may reach when he employs a language brought to perfection; he is read with the same interest in all ages; and time only adds to his reputation by the vain efforts of those who try to imitate him.

The sciences, on the contrary, without bounds like nature herself, increase infinitely by the labours of successive generations the most perfect work; by raising them to a height from which they can never again descend, gives birth to new discoveries which produce in their turn

new works which efface the former from which they originated. Others will present in a point of view more general and more simple, the theories described in the *Principia*, and all the truths which it has displayed; but it will remain as an eternal monument of the profundity of that genius which has revealed to us the greatest law of the universe.

This work and the equally original treatise by the same author on optics, have still the merit of being the best models which he proposed in the sciences, and in the delicate art of making experiments and submitting them to calculation. We there see the most beautiful applications of the method which consists in tracing the principal phenomena to their causes by a succession of inductions, and afterwards to redescend from these causes, to all the details of the phenomena.

General laws are impressed in all individual cases, but they are complicated with so many extraneous circumstances, that the greatest address is often necessary

to develope them. The phenomena most proper for this object must be chosen, they must be multiplied that the attendant circumstances may be varied, and that whatever they have in common may be observed.

We thus ascend successively to relations more and more extended, and we arrive at length at general laws, which are verified either by proofs or by direct experiment, if that is possible, or by examining if they satisfy all the known phenomena.

This is the most certain method by which we can be guided in the search of truth. No philosopher has adhered more faithfully to this method than Newton; it conducted him to his discoveries in analysis, and it led him to the principle of universal gravitation, and to the properties of light. Other philosophers in England, cotemporaries of Newton, adopted it by his example, and it was the base of a great number of excellent works which then appeared.

The philosophers of antiquity following

a contrary path, and considering themselves as at the source of every thing, imagined general causes to explain them.

Their method, which was only productive of vain systems, had not greater success in the hands of Descartes. In the time of Newton, Leibnitz, Malebranche and other philosophers employed it with as little advantage.

At length the inutility of the hypotheses to which it led its followers, and the progress for which the sciences are indebted to the method of inductions has brought back all philosophers to this last method, which Chancellor Bacon has established with the whole force of reason and eloquence, and which Newton has yet more strongly recommended by his discoveries.

It is by means of synthesis that this great geometrician has explained his theory of the system of the world. It appears, however that he found the greater part of his theorems by analysis, the limits of which he has considerably extended, and to which he allows himself to have owed his

general results on the quadratures of curves.

But his great predilection for synthesis, and his esteem for the geometry of the ancients, has induced him to represent his theorems, and even his method of fluxions under a synthetic form. And it is evident by the rules and examples which he has given of these transformations in many works, how much importance he attached to it. We may regret with the geometricians of his time, that he has not followed in the exposition of his discoveries, the path by which he arrived at them; and that he has suppressed the demonstration of many results, such as the equation of the solid of least resistance, preferring the pleasure of leaving it to be divined to that of enlightening his readers.

The knowledge of the method which has guided a man of genius is not less serviceable to the progress of the sciences, and even to his own glory, than his discoveries; and the principal advantage

which has been derived from the famous dispute between Newton and Leibnitz, concerning the invention of the infinitesmal calculus, has been to make known the path of these two great men, in their first analytical labours.

The preference of Newton for the synthetical method, may be explained by the elegance with which he connected his theory of curvilinear motion, with the investigations of the ancients on the conic sections, and the beautiful discoveries which Huygens had published according to this method. Geometrical synthesis has besides the property of never losing sight of its object, and of enlightening the whole path which leads from the first axioms to their last consequences, while algebraic analysis soon makes us forget the principal object, to occupy ourselves with abstract combinations, and it is only at the end that it brings us back to it. But in thus quitting the object of investigation, after having assumed what is in-

dispensably necessary to arrive at the required result, by directing all our attention to the operations of analysis and reserving all our forces to overcome the difficulties which present themselves, we are conducted by the universality of this method, by the inestimable advantage of thus transferring the train of reasoning in mechanical questions, to results often inaccessible to synthesis. The theory of the system of the world offers a great number of examples of this power of analysis to which this theory owes a degree of perfection which it would never have acquired if no other path had been followed than that traced by Newton. Such is the fecundity of analysis, that if we translate particular truths into this universal language, we shall find a number of new and unexpected truths arise merely from the form of expression. No language is so susceptible of the elegance which arises from the developement of a long train of expressions connected with each other, and all flowing from the

same fundamental idea. Analysis unites to all these advantages that of always being able to conduct us to the most simple methods. Nothing more is requisite than to apply it in a convenient manner by a judicious choice of unknown quantities, and by giving to the results the form most easily reducible to geometrical construction, or to numerical calculation. The geometricians of this century, convinced of its superiority, have principally applied themselves to extend its domain, and enlarge it boundaries.

However, geometrical considerations ought not to be abandoned; they are of the greatest utility in the arts. Besides it is curious to imagine the different results of analysis represented in space; and reciprocally, to read all the affections of lines and surfaces, and all the variations in the motions of bodies, in the equations which express them. This approximation of geometry and analysis, diffuses a new light over the sciences; the intellectual operations of the latter, rendered per-

ceptible by the images of the former, are more easy to comprehend, and more interesting to pursue; and when observation realizes these images, and transforms these geometrical results into laws of nature, and when these, embracing the whole universe, display to our view its present and future state, the view of this sublime spectacle, presents to us one of the most noble pleasures reserved for mankind.

About fifty years have passed since the discovery of the theory of gravitation, without any remarkable addition to it. All this time has been requisite for this great truth to be generally understood, and to surmount the obstacles opposed to it by the system of vortices, and the authority of geometricians cotemporary with Newton, who combatted it perhaps from vanity, but who have nevertheless accelerated its progress by their labours on infinitesmal analysis.

At length, their successors have con-

ceived the fortunate idea of applying this analysis to the celestial motions by reducing them to differential equations which they have rigorously integrated, or by converging approximation. They have thus explained by the law of gravitation all the known phenomena of the system of the world, and have given an unhoped for precision to astronomical tables. It has been necessary, for this object, to bring to perfection at once mechanics, optics, and analysis, which principally owe their rapid improvements to their being necessary to the purposes of physical astronomy. It might be rendered yet more correct and simple, but posterity will no doubt see with gratitude that the geometricians of this century have transmitted no astronomical phenomenon of which they have not determined the cause and the law.

Justice to France requires us to observe that if England had the advantage of giving birth to the discovery of universal

gravitation, it is principally to the French geometricians, and to the encouragements of the Academy of Sciences, that the numerous developments of this discovery are due, and the revolution which it has produced in astronomy.

CHAP. VI.

Considerations on the System of the Universe, and on the Future Progress of Astronomy.

LET us now direct our attention to the arrangement of the solar system, and its relation with the stars. The immense globe of the Sun, the focus of these motions, revolves upon its axis in twenty-five days and a half. Its surface is covered with an ocean of luminous matter, whose active effervescence forms variable spots, often very numerous, and sometimes larger than the Earth. Above this ocean exists an immense atmosphere, in which the planets, with their satellites, move, in orbits nearly circular, and in planes little inclined to the ecliptic. Innumerable comets, after having approached the Sun, remove to distances, which evince that his empire

extends beyond the known limits of the planetary system. This luminary not only acts by its attraction upon all these globes, and compels them to move around him, but imparts to them both light and heat; his benign influence gives birth to the animals and plants which cover the surface of the Earth, and analogy induces us to believe, that it produces similar effects on the planets; for, it is not natural to suppose that matter, of which we see the fecundity, develope itself in such various ways, should be sterile upon a planet so large as Jupiter, which, like the Earth, has its days, its nights, and his years, and on which observation discovers changes that indicate very active forces. Man, formed for the temperature which he enjoys upon the Earth, could not, according to all appearance, live upon the other planets; but ought there not to be a diversity of organization suited to the various temperatures of the globes of this universe? If the difference of elements and climates alone, causes such variety in the produc-

tions of the Earth, how infinitely diversified must be the productions of the planets and their satellites? The most active imagination cannot form any just idea of them, but still their existence is extremely probable.

However arbitrary the system of the planets may be, there exists between them some very remarkable relations, which may throw light on their origin; considering them with attention, we are astonished to see all the planets move round the Sun from west to east, and nearly in the same plane, all the satellites moving round their respective planets in the same direction, and nearly in the same plane with the planets. Lastly, the Sun, the planets, and those satellites in which a motion of rotation have been observed, turn on their own axis, in the same direction, and nearly in the same plane as their motion of projection.

A phenomenon so extraordinary, is not the effect of chance, it indicates an universal cause, which has determined all

these motions. To approximate somewhat to the probable explanation of this cause, we should observe that the planetary system, such as we now consider it, is composed of seven planets, and fourteen satellites. We have observed the rotation of the Sun, of five planets, of the Moon, of Saturn's ring, and of his farthest satellite; these motions with those of revolution, form together thirty direct movements, in the same direction. If we conceive the plane of any direct motion whatever, coinciding at first with that of the ecliptic, afterwards inclining itself towards this last plane, and passing over all the degrees of inclination, from zero to half the circumference; it is clear that the motion will be direct in all its inferior inclinations to a hundred degrees, and that it will be retrograde in its inclination beyond that; so that, by the change of inclination alone, the direct and retrograde motions of the solar system, can be represented. Beheld in this point of view, we may reckon twenty-nine motions, of which

the planes are inclined to that of the Earth, at most $\frac{1}{4}$th of the circumference; but, supposing their inclinations had been the effect of chance, they would have extended to half the circumference, and the probability that one of them would have exceeded the quarter, would be $1 - \frac{1}{2^{29}}$, or $\frac{536870911}{536870912}$. It is then extremely probable, that the direction of the planetary motion is not the effect of chance, and this becomes still more probable, if we consider that the inclination of the greatest number of these motions to the ecliptic, is very small, and much less than a quarter of the circumference.

Another phenomenon of the solar system equally remarkable, is the small excentricity of the orbits of the planets and their satellites, while those of comets are much extended. The orbits of the system offer no intermediate shades between a great and small excentricity. We are here again compelled to acknowledge the effect of a regular cause; chance alone could not have given a form nearly circu-

lar, to the orbits of all the planets. This cause then must also have influenced the great excentricity of the orbits of comets, and what is very extraordinary, without having any influence on the direction of their motion; for, in observing the orbits of retrograde comets, as being inclined more than 100° to the ecliptic, we find that the mean inclination of the orbits of all the observed comets, approaches near to 100°, which would be the case if the bodies had been projected at random.

Thus, to investigate the cause of the primitive motions of the planets, we have given the five following phenomena: 1st, The motions of planets in the same direction, and nearly in the same plane. 2d, The motion of their satellites in the same direction, and nearly in the same plane with those of the planets. 3d, The motion of rotation of these different bodies, and of the Sun in the same direction as their motion of projection, and in planes but little different. 4th, The small excentricity of the orbits of the planets, and

of their satellites. 5th, The great excentricity of the orbits of comets, although their inclinations may have been left to chance.

Buffon is the only one whom I have known, who, since the discovery of the true system of the world, has endeavoured to investigate the origin of the planets, and of their satellites. He supposes that a comet, in falling from the Sun, may have driven off a torrent of matter, which united itself at a distance, into various globes, greater or smaller, and more or less distant from this luminary. These globes are the planets and satellites, which, by their cooling, are become opaque and solid.

This hypothesis accounts for the first of the five preceding phenomena; for, it is clear that all bodies thus formed, must move nearly in the plane which passes through the centre of the Sun, and in the direction of the torrent of matter which produces them. The four other phenomena appears to me inexplicable by his

theory. In fact, the absolute motion of the particles of a planet would then be in the same direction of the motion of its centre of gravity; but it does not follow that the rotation of the planet would be in the same direction. Thus, the Earth may turn from west to east, and yet the absolute direction of each of its particles may be from east to west. What I say of the rotatory motion of the planets, is equally applicable to the motion of their satellites in their orbits, of which the direction in the hypothesis he adopts, is not necessarily the same with the projectile motion of the planets.

The small excentricity of the motion of the planetary orbits, is not only very difficult to explain on this hypothesis, but the phenomenon contradicts it. We know by the theory of central forces, that if a body moving in an orbit round the Sun, touched the surface of this luminary, it would uniformly return to it at the completion of each revolution, from whence it follows, that if the planets had originally

been detached from the Sun, they would have touched it at every revolution, and their orbits, far from being circular, would be very excentric. It is true, that a torrent of matter, sent off from the Sun, cannot correctly be compared to a globe which touches its surface. The impulse which the particles of this torrent receive from one another, and the reciprocal attraction exercised among them, may change the direction of their motion, and increase their perihelion distances; but their orbits would uniformly become very excentric, or at least it must be a very extraordinary chance that would give them excentricities so small as those of the planets. In a word, we do not see, in this hypothesis of Buffon, why the orbits of about eighty comets, already observed, are all very elliptical. This hypothesis, then, is far from accounting for the preceding phenomena. Let us see if it is possible to arrive at their true cause.

Whatever be its nature, since it has produced or directed the motion of the

planets and their satellites, it must have embraced all these bodies, and considering the prodigious distance which separates them, they can only be a fluid of immense extent. To have given in the same direction, a motion nearly circular round the Sun, this fluid must have surrounded the luminary like an atmosphere. This view, therefore, of planetary motion, leads us to think, that in consequence of excessive heat, the atmosphere of the Sun originally extended beyond the orbits of all the planets, and that it has gradually contracted itself to its present limits, which may have taken place from causes similar to those which caused the famous star that suddenly appeared in 1572, in the constellation Cassiopæa, to shine with the most brilliant splendour during many months.

The great excentricity of the orbits of comets, leads to the same result; it evidently indicates the disappearance of a great number of orbits less excentric, which indicates an atmosphere round the Sun, extending beyond the perihelion of

observable comets, and which, in destroying the motion of those which they have traversed in a duration of such extent, have re-united themselves to the Sun. Thus, we see that there can at present only exist such comets as were beyond this limit at that period. And as we can observe only those which in their perihelion approach near the Sun, their orbits must be very excentric: but, at the same time, it is evident that their inclinations must present the same inequalities as if the bodies had been sent off at random, since the solar atmosphere has no influence over their motions. Thus, the long period of the revolutions of comets, the great excentricity of their orbits, and the variety of their inclinations, are very naturally explained by means of this atmosphere.*

But how has it determined the motions of revolution and rotation of the planets? If these bodies had penetrated this fluid, its resistance would have caused them to fall into the Sun. We may then conjecture, that they have been formed at the

successive bounds of this atmosphere, by the condensation of zones, which it must have abandoned in the plane of its equator, and in becoming cold have condensed themselves towards the surface of this luminary, as we have seen in the preceding Book. One may likewise conjecture, that the satellites have been formed in a similar way by the atmosphere of the planets. The five phenomena, explained above, naturally result from this hypothesis, to which the rings of Saturn add an additional degree of probability.

Whatever may have been the origin of this arrangement of the planetary system, which I offer with that distrust which every thing ought to inspire that is not the result of observation or calculation; it is certain that its elements are so arranged, that it must possess the greatest stability, if foreign observations do not disturb it. Through this cause alone, that the motions of planets and satellites are nearly circular, and impelled in the same direction, and in planes differing but

little from each other, it arises that this system can only oscillate to a certain extent, from which its deviation must be extremely limited; the mean motions of rotation and revolution of these different bodies are uniform, and their mean distances to the foci of the principal forces which animate them, are uniform. It seems that nature has disposed every thing in the heavens to insure the duration of the system by views similar to those which she appears to us so admirably to follow upon Earth, to preserve the individual and insure the perpetuity of the species.

Let us now look beyond the solar system. Innumerable suns, which may be the foci of as many planetary systems, are spread out in the immensity of space, and at such a distance from the Earth, that the entire diameter of it, seen from their centre, is insensible. Many stars experience both in their colour and splendour, periodical variations, very remarkable; there are some which have appeared all at once, and disappeared after having for some

time spread a brilliant light. What prodigious change must have operated on the surface of these great bodies, to be thus sensible at the distance which separates them from us, and how much they must exceed those which we observe on the surface of the Sun? All these bodies which are become invisible, remain in the same place where they were observed, since there was no change during the time of their appearance, there exist then in space obscure bodies as considerable, and perhaps as numerous as the stars. A luminous star, of the same density as the Earth, and whose diameter should be two hundred and fifty times larger than that of the Sun, would not, in consequence of its attraction, allow any of its rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe, may, through this cause, be invisible. A star, which, without being of this magnitude, should yet considerably surpass the Sun, would perceptibly weaken the velo-

city of its light, and thus augment the extent of its aberration. This difference in the aberration of stars and their situation, observed at the moment of their transient splendor, the determination of all the changeable stars, and the periodical variations of their light; in a word, the motions peculiar to all those great bodies, which, influenced by their mutual attraction, and probably by their primitive impulses, describe immense orbits, should, relatively to the stars, be the principal objects of future astronomy.

It appears that these stars, far from being disseminated at distances nearly equal in space, are united in various groups, each consisting of many millions of stars. Our Sun, and the most brilliant stars, probably make part of one of these groups, which, seen from the point where we are, seems to encircle the heavens, and forms the milky way. The great number of stars which are seen at once in the field of a large telescope, directed towards this

way, proves its immense depth, which surpasses a thousand times the distance of Sirius from the Earth; as it recedes, it terminates, by presenting the appearance of a white and continued light, of small diameter, for then, the irradiation which exists even in the most powerful telescopes, covers and obscures the intervals between the stars. It is then probable, that those nebulæ, without distinct stars, are groups of stars seen from a distance, and which, if approached, would present appearances similar to the milky way.

The relative distances of the stars which form each group, are at least a hundred thousand times greater than the distance of the Sun from the Earth. Thus, we may judge of the prodigious extent of these groups, by the number of stars which are perceived in the milky way, if we afterwards reflect on the small extent and infinite number of nebulæ which are separated from one another by an interval incomparably greater than the relative

distance of the stars of which they are formed; the imagination, lost in the immensity of the universe, will have difficulty to conceive its bounds.

From these considerations, founded on telescopic observations, it follows, that nebulæ, which appear so well defined, that their centres can be precisely determined, are, with regard to us, the celestial objects most fixed, and those to which it is best to refer the situation of all the stars. It follows then, that the motions of the bodies of our solar system are very complicated. The Moon describes an orbit nearly circular around the Earth; but, seen from the Sun, she describes a series of epicycloids, of which the centres are on the circumference of the terrestrial orbit. In like manner, the Earth describes a series of epicycloids, of which the centres on the arch which the Sun describes around the centre of gravity of our nebulæ; finally, the Sun himself describes a series of epicycloids, of which the centres are on the arch described by the centre of gra-

vity of our nebulæ around that of the universe. Astronomy has already made one great step in making us acquainted with the motion of the Earth, and the series of epicycles which the Moon and the satellites describe upon the orbits of the planets. It remains to determine the orbit of the Sun, and the centre of gravity of its nebulæ; but, if ages are necessary to become acquainted with the motions of the planetary system, what a prodigious duration of time will it require to determine the motions of the Sun and stars? Observation begins to render them perceptible; an attempt has been made to explain them by a change of position in the Sun, indicated by its rotatory motion. Many observations are sufficiently well explained, by supposing the solar system carried towards the constellation Hercules. Other observations seem to prove, that these apparent motions of the stars are a combination of their real motion, with that of the Sun. Upon this subject, time will discover curious and important facts.

There still remains numerous discoveries to be made in our own system. The planet Uranus and its satellites, but lately known to us, leaves room to suspect the existence of other planets, hitherto unobserved. We cannot yet determine the rotatory motion, or the flattening of many of the planets, and the greatest part of their satellites. We know not, with sufficient precision, the density of all these bodies. The theory of their motions is a series of approximations, whose convergence depends, at the same time, on the perfection of our instruments, and the progress of analysis, and which must, by these means, daily acquire new degrees of correctness. By accurate and repeated measurement, the inequalities in the figure of the Earth, and the variation of weight on its surface, will be determined. The return of comets, already observed, new comets which will appear, the appearance of those, which, moving in hyperbolic orbits, can wander from system to system,

the disturbance all those stars experience, and which, at the approach of a large planet, may entirely change their orbits, as is conjectured, happened by the action of Jupiter on the comet of 1770; the accidents, that the proximity, and even the shock of these bodies, may occasion in the planets, and in the satellites; in a word, the changes which the motions of the solar system experience, with respect to the stars; such are the principal objects which the system presents to astronomical researches, and future geometricians.

Contemplated as one grand whole, astronomy is the most beautiful monument of the human mind; the noblest record of its intelligence. Seduced by the illusions of the senses, and of self-love, man considered himself, for a long time, as the centre of the motion of the celestial bodies, and his pride was justly punished by the vain terrors they inspired. The labour of many ages has at length withdrawn the

veil which covered the system. Man appears, upon a small planet, almost imperceptible in the vast extent of the solar system, itself only an insensible point in the immensity of space. The sublime results to which this discovery has led, may console him for the limited place assigned him in the universe. Let us carefully preserve, and even augment the number of these sublime discoveries, which form the delight of thinking beings.

They have rendered important services to navigation and astronomy; but their great benefit has been the having dissipated the alarms occasioned by extraordinary celestial phenomena, and destroyed the errors springing from the ignorance of our true relation with nature; errors so much the more fatal, as social order can only rest in the basis of these relations. *Truth, Justice*; these are its immutable laws. Far from us be the dangerous maxim, that it is sometimes useful to

mislead, to deceive, and enslave mankind; to insure their happiness. Cruel experience has at all times proved that with impunity, these sacred laws can never be infringed.

FINIS.

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